



MULTI-AXIS CNC MACHINING AND TOOLPATH STRATEGIES FOR COMPLEX FREE-FORM GEOMETRIES: A SYSTEMATIC REVIEW OF GEOMETRY, KINEMATICS, PROCESS PHYSICS AND INTELLIGENT OPTIMIZATION

Trust M. Ncube, Gratitude Muchandiona, Tanatswa Mufudza and Martin Banda *

Department of Industrial and Manufacturing Engineering, National University of Science and Technology, P.O. Box AC 939, Ascot, Bulawayo, Zimbabwe.

* Corresponding Author

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ABSTRACT

In addition to offering the enabling process for the turbine blade, blisks, impellers, moulds, dies and other free-form components, multi-axis computer numerical control (CNC) machining introduces the geometry of toolpath, kinematics of machine, cutting mechanics, surface integrity and energy use. In this paper, research on five-axis and multi-axis toolpath strategies for complex geometries is summarized and critically analyzed, and the circumstances under which the various strategies are viable are identified. The literature was searched for in a systematic manner using the PRISMA 2020 reporting logic on studies published from 2016 to 2026. A total of fifty one primary studies and two previous reviews were coded into eight themes and summarized thematically at six interacting decision layers: geometry, toolpath generation, tool orientation, machine kinematics, process behaviour and optimisation. Fifteen research and development process optimisation, maintenance, Industry 4.0 and capability studies were conducted by the authors' research group and used to interpret the transferability into resource-constrained manufacturing contexts. There is no one dominant toolpath family throughout the categories of accuracy, cycle time, and stability and surface integrity. The most reliable techniques for controlling residual height are the "isoscalops" and "partitions" approaches, the "flank" approach and "trochoidal" or "barrel" (non-spherical tool) approach have advantages in terms of productivity or accessibility for appropriate geometry. On the part surface, an efficient path may not be efficient when generated via inverse kinematics, as singularities, rotary-axis reversals and feedrate limits produce a different realised path. Planning has been recently associated with models of cutting forces, digital twins, machine learning, reinforcement learning, but the connection of planning with industrial closed-loop validation, the generalisation across machines, and the consideration of energy and residual stress as planning goals are limited. The review describes a six-layer taxonomy and a coupled decision model for multi-axis planning, characterizes the corpus according to year and theme, and outlines a research agenda for the machine-aware, physics-informed and energy-aware computer-aided manufacturing (CAM) and the adoption of the technology in the development of industrial economies.

Keywords: Five-Axis Machining; Multi-Axis CNC; Toolpath Planning; Tool Orientation; Machine Kinematics; Surface Integrity; Digital Twin; Energy Efficiency; Systematic Review

1 INTRODUCTION

In today's engineering world, geometries that aren't easily machined with fixed axis are becoming greater by the day. Continuously varying curvature, narrow passages and deep features characterize turbine blades, blisks, impellers, moulds, dies and orthopaedic or aerospace parts. In the case of such parts, the five-axis machine can move the cutter and change its orientation, which reduces interference, and shortens the effective tool overhang and enables more of the part to be finished in a single set-up (Lasemi, Xue and Gu, 2010; Mali, Gupta and Ramkumar, 2021). The same ability can be used, however, to make the toolpath planning a coupled problem. The definition of a toolpath is extended from a spatial curve to the variable orientation of the tool which is directly actuated to control the rotary axes of the machine (Liang, Kang and Fang, 2021).

The impact of this coupling is felt directly on the aspects of accuracy and productivity. Cripps et al. (2017) demonstrated that kinematic singularities could cause sudden and unwanted axis movement in 5 axis machines and Zhang et al. (2018) implemented geometric constraints to prevent the singular regions. Huang, Lu and Zhu (2018) showed that the feedrate has to be programmed for both linear and angular motion. The studies indicate that a common engineering problem is that a good path on the part surface may not lead to a good path on the machine. Such capability is not only needed in the aerospace sector. Accurate machining of complex surfaces is important for free form aerofoil blades for small vertical axis wind turbines (VAWTs) (Muyambo et al., 2025), lightweight load-bearing mechanisms after topology optimisation (Tandi et al., 2025), assistive devices frames made of aluminium (Munotyaani et al., 2026) and the cavities of injection moulds whose surface quality determines shrinkage and warpage of the moulded part (Mapindu et al., 2026a).

Meanwhile, Zimbabwe manufacturers' surveys reveal that NC machines, rapid prototyping machines and simulation software are still in limited and low usage and application (Muhla et al., 2023; Muhla et al., 2025a). Multi-axis machining is thus a frontier research topic and a skills development challenge to be addressed in the development of industrial economies, and learning factories have been suggested as one way to address this challenge (Ndiyamba et al., 2026). Parts of this field have been reviewed. The research of Mali, Gupta and Ramkumar (2021) covered a decade of free-form surface milling and of Liang, Kang and Fang (2021) covered tool-orientation planning. None was developed to link the geometry of the toolpath, orientation, machine kinematics, process physics, surface integrity, energy and data-driven optimisation in one analytical model, and there has been no synthesis of the literature on the development of digital twins, reinforcement learning and convex orientation optimisation since 2021. This review fills the void.

Aim, research questions and contributions

The aim of this review is to critically synthesise research on multi-axis CNC machining and toolpath strategies for complex geometries, with emphasis on the relationships between toolpath generation, tool orientation, machine behaviour and machining performance. Four research questions (RQs) guided the review:

- **RQ1:** Which toolpath strategies and tool-orientation methods have been developed for five-axis and multi-axis machining of complex geometries since 2016?
- **RQ2:** How do current strategies address geometric accuracy, collision avoidance, machine kinematics, machining time, cutting stability and surface integrity?
- **RQ3:** Which optimisation, modelling and intelligent-manufacturing methods are used to improve multi-axis machining performance?
- **RQ4:** Which research gaps remain in integrating geometry, machine behaviour, process physics, sustainability and real-time feedback?

The paper makes four contributions: (i) a PRISMA-structured map of the 2016–2026 literature by year and theme; (ii) a six-layer taxonomy that locates each study by the planning decision it addresses; (iii) a cross-study synthesis of agreements and tensions, including a coupled decision model for multi-axis planning; and (iv) a research agenda that also considers transfer to resource-constrained manufacturing environments. Section 2 sets out the technical background, Section 3 the review method, Section 4 the descriptive results and taxonomy, Section 5 the thematic synthesis, Section 6 the discussion, Section 7 the research agenda and Section 8 the conclusions.

2 BACKGROUND AND THEORETICAL FOUNDATION

2.1 Five-axis machine configurations and toolpath representation

Three axis milling is the process in which the cutting tool axis remains stationary and the cutter centre path is programmed. The five-axis machine has two rotational degrees of freedom which can change the direction of the tool axis as the cutter traverses the workpiece surface. There are three different methods for the two rotary axes to be

carried; (a) carried by the rotary axis of the workpiece (table–table); (b) carried by both the rotary axis of the spindle and the table (head–table); and (c) carried by the rotary axis of the spindle (head–head) as shown in Figure 1. The configuration influences the workspace, the position of the singularities and the stiffness of the kinematic chain, hence the realisation of a given toolpath by the machine (Bohez, 2002).

The position vector and a unit vector of the tool axis for a cutter location point. For CAM systems, the tool axis is typically represented as a local axis at the cutter-contact point with the surface normal and the feed direction, and a lead (inclination) angle about the cross-feed direction and a tilt (yaw) angle about the feed direction as shown in Figure 1(d) (Liang, Kang and Fang, 2021). The inverse kinematics then transforms each position–orientation pair into linear and rotary axis commands. This is a non-linear mapping and a small change of the orientation around a singular configuration may result in a large movement of the rotary axis (Cripps et al., 2017).

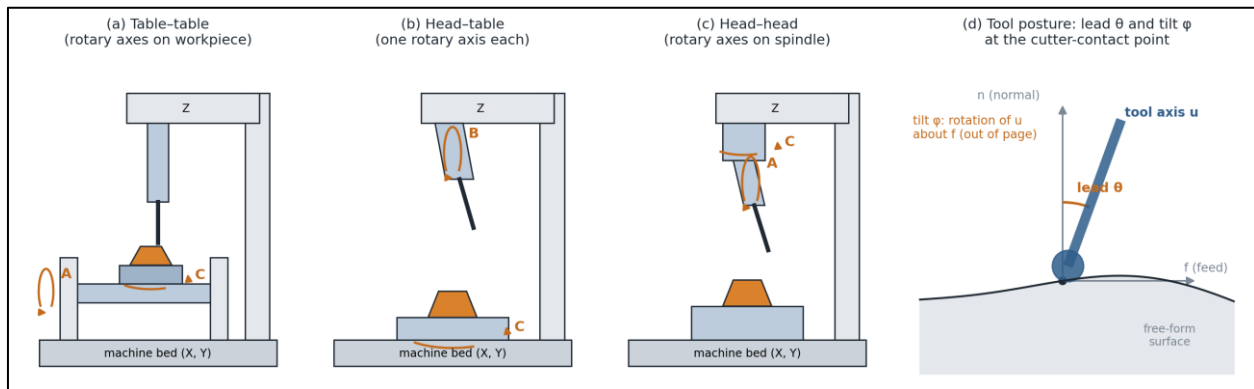


Figure 1: Common five-axis kinematic configurations, (a) table–table, (b) head–table and (c) head–head, and (d) the lead–tilt description of tool posture at the cutter-contact point. Panels (a)–(c) are adapted from the kinematic-chain classification of Bohez (2002); panel (d) is adapted from the tool-orientation conventions summarised by Liang, Kang and Fang (2021). Redrawn by the authors.

2.2 Major toolpath families

The reviewed studies can be classified in the context of various toolpath families. Iso-parametric and iso-planar paths are simple and still widely-used in commercial CAM but result in an uneven scallop height on strongly curved surfaces. In response to this, iso-scallop methods adjust the side step to keep the residual cusp height constant (Song et al., 2025). Region based or partition based methods are methods that partition the surface based on the geometry or the machining properties, and then apply an appropriate pattern in each region (Xu and Li, 2019; Li and Tang, 2021). The side of the cutter is used for milling the ruled or near-developable surfaces in few passes, which is known as flank (swarf) milling (Chu, Chen and Chang, 2020). Trochoidal strategy is based on the idea of introducing a circular motion in addition to the feed motion in slots and deep cavities to reduce radial engagement (Li et al., 2020). A barrel, conical and other non-spherical tools can enhance the effective cutting width, but make orientation planning more challenging (Lu et al., 2019; Suzuki, Okamoto and Morishige, 2021; Hao et al., 2026).

2.3 Machine kinematics and process physics

The machine does not move the toolpath in the coordinate system where the CAM algorithm is creating a toolpath. Realised motion is changed by axis limits, acceleration, jerk, singularities and controller interpolation. Huang, Lu and Zhu (2018) solved this by simultaneously planning the linear and angular trajectory, and Tajima and Sencer (2019) presented an accurate real-time interpolation method using local corner smoothing.

Process physics is added on top of that. The cutting forces and the stability limits are not constant in the direction of tool engagement, and these vary with the orientation, feed direction and surface curvature. Berglind, Plakhotnik and Ozturk (2017) modelled a discretised force for arbitrary engagement and feed direction, and Song et al. (2019) extended the mechanistic force modelling to five-axis milling. These models correlate CAM geometry with the forces that are actually produced while machining. The experimental studies at the three-axis levels validate that the dominant process variable varies with the response; in CNC milling of mild steel, the feed rate was the dominant process variable at 82.81% variation in surface roughness, whereas the spindle speed was the dominant process variable at 63.40% variation in flank wear (Mapindu et al., 2026b). These effects are further complicated by the additional variable of orientation in five axis machining.

2.4 Surface quality, error and energy

Geometric tolerance isn't the only factor in assessing surface quality. The surface is influenced by tool orientation, engagement, direction of feed and motion of the machine (Wang et al., 2020; Fu et al., 2023). Subsurface residual stresses may still be present on a surface that meets the roughness requirement, which can impact fatigue performance (Zhao et al., 2023; Wang et al., 2023). The geometric errors of the machine also cause propagation errors to the formed part, and the accuracy is not only influenced by the programmed geometric accuracy, but also by the geometric accuracy of the machine (Liu et al., 2018; Xiang et al., 2018; Fu et al., 2018; Sato and Shirase, 2018).

Energy is now a part of the machining decision. Xu, Luo and Tang (2016) developed energy-saving toolpaths for five-axis end milling and Vishnu, Varghese and Gurumoorthy (2020) predicted energy during five-axis process planning from data. Experimental investigation of honing of cast-iron cylinder liners (Tayisepi et al., 2024) and planing of teak, saligna and pine (Tayisepi et al., 2023) revealed that the parameter settings that lead to the minimum energy are not the same as those that lead to the minimum roughness, and that there is a trade-off between them that depends on material. When feedrate, rotary axis motion and machine power are taken into account, the shortest geometric path is not necessarily the lowest energy path.

3 REVIEW METHODOLOGY

The review was conducted as a systematic literature review (SLR) instead of a narrative review. It adheres to the reporting guidelines of PRISMA 2020 (moore, 2021), which include transparency in reporting information sources, search strategies, study selection and synthesis (Page et al., 2021). The engineering emphasis was maintained throughout to ensure that methods were compared on their machining behaviour, rather than by the number of publications.

3.1 Information sources and search scope

The major information sources used were Google Scholar as it indexes the main publishers in the field (Elsevier, Springer, Taylor & Francis, SAGE, ASME, IEEE and Fuji Technology Press) as well as conference proceedings. Two previous reviews (Liang, Kang and Fang, 2021; Mali, Gupta and Ramkumar, 2021) and highly cited primary studies were followed up on both backward and forward to minimise the possibility of missing out on relevant work.

3.2 Search terms and Boolean strategy

These were combined into three concept groups for the search: the machining process, toolpath or orientation planning, and complex geometry or performance outcomes. A typical search string was: ("five-axis machining" OR "5-axis machining" OR "multi-axis machining" OR "multiaxis machining") AND ("toolpath" OR "tool path" OR "tool orientation" OR "feedrate" OR "cutter location") AND ("free-form surface" OR freeform OR blisk OR impeller OR turbine OR "complex geometry"). A second string was related to surface roughness, surface integrity, chatter, cutting force, energy, machining error, cycle time and optimisation.

3.3 Eligibility criteria and study selection

Record de-duplication and screening by title and abstract and then full-text screening against the criteria in Table 1 were performed. Studies were considered if they directly addressed the generation of toolpaths for five-axis or multi-axis milling or addressed tool orientation, machine kinematics, process stability, surface quality, intelligent optimisation or energy optimisation of the process. Studies that did not include any relevance to complex-geometry machining, as well as non-peer-reviewed commercial material and papers, were not included. The selection process is briefly summarised in Figure 2.

Table 1: Eligibility criteria used to define the review scope

Criterion	Included	Excluded
Publication period	2016–2026; a small number of earlier seminal sources where needed to explain fundamentals	Outside the period without a clear methodological reason
Source type	Peer-reviewed journal articles and peer-reviewed proceedings	Blogs, commercial pages, theses and non-reviewed summaries
Technical scope	Five-axis or multi-axis CNC milling: toolpaths, orientation, kinematics, stability, surface quality, energy and optimisation	Purely three-axis work with no transferable relevance

Geometry	Free-form surfaces, blisks, impellers, ruled surfaces, deep cavities and other complex forms	Studies unrelated to complex-geometry machining
Evidence	Analytical modelling, simulation, numerical optimisation or machining experiment	Papers without sufficient technical description
Language	English-language scholarly literature	Sources that could not be reliably interpreted or verified

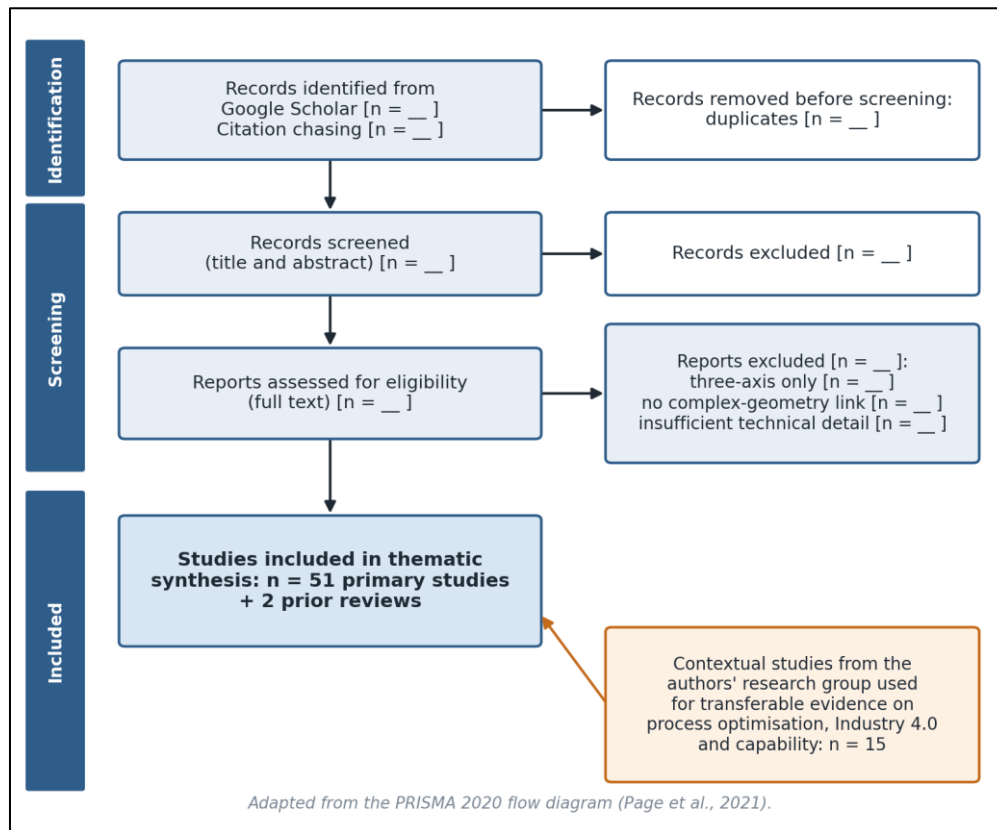


Figure 2: PRISMA 2020 flow diagram of study identification, screening and inclusion. Adapted from Page et al. (2021). Bracketed counts are to be completed from the audited search log.

3.4 Quality appraisal

These four questions were applied in each of the studies included: (Q1) Does the method described in the study provide sufficient technical detail to reproduce? (Q2) Is the performance measure suitable to the stated objective? (Q3) Is the method validated by simulation or machining experiment or both? (Q4) Do the results of the study report the performance measure sufficiently so that it could be compared to other studies? Experimental validation was particularly given importance as a mathematically elegant toolpath does not necessarily qualify as a pragmatic solution to manufacturing. Studies were not excluded due to appraisal scores; appraisal was used to inform the weight of each finding in the synthesis.

3.5 Data extraction and synthesis

The following factors were extracted from each study: geometry type, machine configuration, tool type, planning decision addressed, method of the study, objective(s), validation type and principal result. Studies were then classified to the main theme (Table 2), and synthesised thematically. Rather than considering each article as a standalone article, the papers were sorted by the engineering problem that it was designed to solve, allowing for comparisons of studies which employed different algorithms and/or test conditions. Examples of orientation optimisation were compared among rotation-minimising, tractrix, feasible-region and convex-optimisation approaches, and process-performance studies were compared between cutting force, chatter, surface integrity, energy and cycle time.

3.6 Contextual evidence from the authors' research group

The authors' research group read 15 studies that were not included in the 5-axis studies, but included in the corpus. They address issues on experimental process optimisation using Taguchi designs, analysis of variance (ANOVA) and grey relational analysis (GRA) in milling, honing, wood planing, injection moulding, fused deposition modelling and edible-oil bleaching, data-driven and agent-based maintenance, Industry 4.0, agility and human-factors survey of manufacturing in Zimbabwe, and product-development case studies, among others. They were employed to examine the generalizability of the findings from the five-axis literature to resource-limited industrial settings, and are presented in Table 4.

4 DESCRIPTIVE RESULTS AND TAXONOMY OF THE LITERATURE

4.1 Distribution of the corpus

This synthesis includes 51 main studies and two previous reviews. Their distribution by year and theme is presented in figure 3. The largest theme (15 studies, 29%) is the generation of toolpath, followed by the kinematics of the machine (7 studies, 14%) and feedrate and set-up (7 studies, 14%) and finally geometric error and measurement (7 studies, 14%). Research for geometric-error is focused in 2018-2019 while the work on intelligent methods and digital twins seems to start from 2022. Only two studies focused on energy, and some others had energy as a secondary focus. The trend suggests that a single geometric problem is shifted to planning that involves several layers of research.

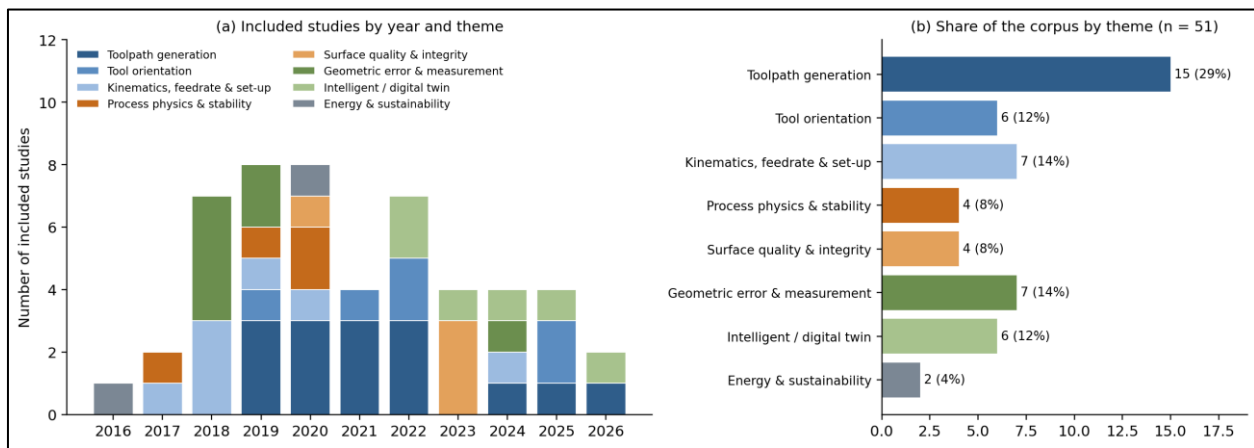


Figure 3: Distribution of the 51 included primary studies (a) by publication year and theme and (b) by theme.
Source: authors' coding of the included studies.

Table 2: Thematic coding of the 51 included primary studies

Theme	n	Representative studies	Principal performance measures
Toolpath generation	15	Xu and Li (2019); Chu, Chen and Chang (2020); Li and Tang (2021); Sun et al. (2022); Bo, Fan and Bartoň (2022); Dai et al. (2024); Song et al. (2025); Hao et al. (2026)	Scallop height, gouging, path length, machining error, efficiency
Tool orientation	6	Zou et al. (2019, 2022); Wang, Xu and Sun (2021); Xiao et al. (2022); Wu et al. (2025); Chen et al. (2025)	Orientation smoothness, rotary-axis motion, contour error, interference
Kinematics, feedrate and set-up	7	Cripps et al. (2017); Zhang et al. (2018); Huang, Lu and Zhu (2018); Yang, Aslan and Altintas (2018); Tajima and Sencer (2019); Gao et al. (2020); Liu et al. (2024)	Singularity avoidance, tracking error, feedrate, jerk, cycle time
Process physics and stability	4	Berglind, Plakhotnik and Ozturk (2017); Song et al. (2019); Li, Kilic and Altintas (2020); Zhan et al. (2020)	Cutting force, dynamics, chatter stability
Surface quality and integrity	4	Wang et al. (2020); Zhao et al. (2023); Wang et al. (2023); Fu et al. (2023)	Roughness, topography, residual stress

Geometric error and measurement	7	Liu et al. (2018); Xiang et al. (2018); Fu et al. (2018, 2019); Sato and Shirase (2018); Li et al. (2019); He et al. (2024)	Error identification, volumetric error, compensation accuracy
Intelligent methods and digital twins	6	Zhao et al. (2022); Rudel et al. (2022); Kukreja and Pande (2023); Chien, Sencer and Ward (2024); Lu et al. (2025); Chichell et al. (2026)	Prediction accuracy, cycle time, roughness stability, energy
Energy and sustainability	2	Xu, Luo and Tang (2016); Vishnu, Varghese and Gurumoorthy (2020)	Machine energy, energy prediction

Note: prior reviews by Liang, Kang and Fang (2021) and Mali, Gupta and Ramkumar (2021) were used for citation chasing and are not counted in n. Lu et al. (2019), Sharma et al. (2019), Zhang et al. (2020), Li et al. (2020, 2021), Suzuki, Okamoto and Morishige (2021) and Yao et al. (2022) complete the toolpath-generation theme.

4.2 A six-layer taxonomy

The existing literature can be broken down into six collated decision layers (Figure 4): part geometry; toolpath generation; tool orientation; machine kinematics; process behaviour and quality; and optimisation and intelligence. Previous contributions tended to focus on one layer, while recent contributions are increasingly associating two or more layers. The taxonomy also incorporates a feedback loop, as measured quality, power and machine data can be fed back upstream to improve the decision-making, such as digital-twin-based parameter tuning (Zhao et al., 2022) and condition-monitoring architectures that share sensor information with diagnostic agents (Mashava et al., 2026).

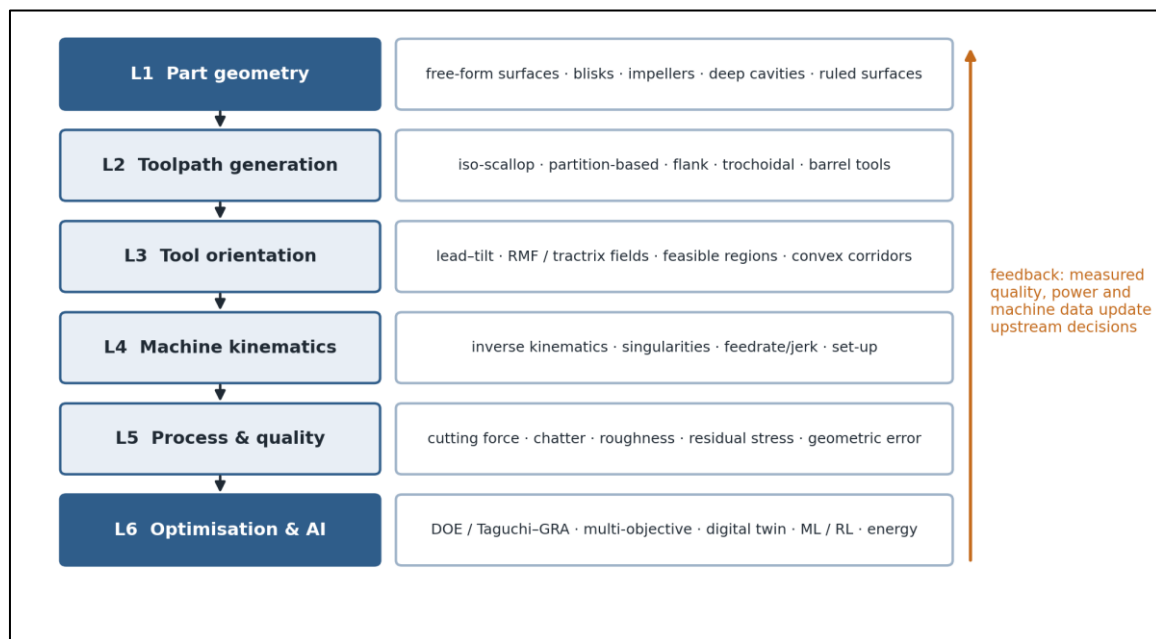


Figure 4: Six-layer taxonomy of multi-axis CNC toolpath research with a feedback path from measured performance to upstream planning decisions. Source: authors' own construction.

5 THEMATIC SYNTHESIS

5.1 Geometry and toolpath generation

Research in geometry is to create paths that move along the part under control of the amount of material removed and the prevention of gouging. Region based methods are a way that reduces a complex surface into regions where different patterns can be used for the machining. Some of them use an image representation of the surface to help with region-based five-axis path generation (Xu and Li, 2019) and partition free-form surfaces so that a non-spherical tool can be efficiently employed in each partition (Li and Tang, 2021). Hao et al. (2026) proposed the isoscallop field along with surface partitioning for non-spherical cutters, while Song et al. (2025) introduced the uniform-scallop and uniform-strip-width constraints for flat and filleted end mills, respectively, and separated the problem into two components: planning under uniform-scallop constraint and uniform-strip-width constraint. The studies together demonstrate that a strategy that works for a set of manageable regions is better than a strategy that is forced throughout the component.

Flank milling is of special significance for ruled and near-developable surfaces. However, reducing the machining error due to path continuity was performed by Chu, Chen and Chang (2020); planning flank paths for free-form surfaces by Zhang et al. (2020); modelling the envelope surface of the conical cutter at a given cutting speed by Sun et al. (2022); and generating flank paths from the measured error distribution by Yao et al. (2022). While all agree that the geometric error is determined by the way the cutter comes into contact with the surface, they differ on the formulation of the contact condition and if measured error is fed back.

The strategies of trochoidal and special tools address accessibility and engagement. Li et al. (2020), Li et al. (2021) have developed five-axis trochoidal flank milling for deep cavity and variable-depth slot, Bo, Fan and Bartoň (2022) have applied the custom-shaped tool to trochoidal flank milling for deep cavities, and Dai et al. (2024) have applied the constant cutting depth method to the blisk milling process. The barrel cutter has been used in blisks (Lu et al., 2019) and in free-form surfaces while maintaining the tool posture continuity (Suzuki, Okamoto and Morishige, 2021). Because single-point methods are unable to be applied at multiple contact points where two Bézier surfaces intersect, Sharma et al. (2019) placed the tool at these points. The common lesson is that wide contact tools are only going to be productive tools if you can keep the changes of engagement and posture within control during the orientation planning process.

5.2 Tool orientation

Geometric planning is a machine-motion problem when considering tool orientation. Access and collision should be provided with an appropriate orientation and/angular motion limited. Zou et al. (2019) constructed orientation fields from rotation-minimising frames and Zou et al. (2022) used a spatial tractrix formulation. Wang, Xu and Sun (2021) optimized the orientation to enhance kinematic performance, while Xiao et al. (2022) introduced a constraint on the contour error without affecting the kinematic performance. Wu et al. (2025) stated that a safe attitude corridor can be defined as a convex optimisation problem, and Chen et al. (2025) designed blisk toolpaths whose orientation varies with respect to kinematic features of rotary axes of a machine. The progression illustrates the evolution from geometric construction to constrained optimisation, incorporating machine behaviour.

5.3 Machine kinematics, feedrate and set-up

One consistent result is that quality of a five-axis tool path is not completely captured by geometry. CAM provides a cutter path, but the machine provides axis motions and after inverse kinematics, a minor change in orientation can be a major change in one of the rotary axes. Cripps et al. (2017) showed that the singularities were important, Zhang et al. (2018) excluded them by the geometric constraints, and Huang, Lu and Zhu (2018) designed linear and angular trajectories simultaneously. Tajima and Sencer (2019) demonstrated that real-time realising of corner smoothing has an impact on accuracy and cycle time.

A second lever that's frequently overlooked is workpiece set-up. The minimum tracking error of the workpiece location on rotary tables has been used to reduce tracking error (Yang et al., 2018), the set-up selection problem has been formulated as a general optimisation problem (Gao et al., 2020), and motion coherence and stiffness have been used to link set-up optimisation (Liu et al., 2024). These contributions transform the question of planning from "Which path follows the surface?" to "Which path and set-up enables the machine to reliably follow the surface."

5.4 Process physics and stability

The same point is made from the process side in cutting-force and stability studies. Berglind et al. (2017) applied a discrete cutting element approach to treat arbitrary engagement and feed directions for milling, whereas Song et al. (2019) created a generalised cutting-force model for 5-axis milling. Li, Kilic and Altintas (2020) extended the modelling work to cutting dynamics while Zhan et al. (2020) expanded the modelling to the stability of ball-end milling using variable-pitch tools. The formulations vary, but they all show that as the tool orientation varies, the engagement is not constant, and that an efficient (geometric) path may be unstable or have a limited maximum engagement. However, high fidelity models are also often very expensive to compute which is the reason they are rarely integrated directly into path optimisation.

5.5 Surface quality and surface integrity

Surface quality is affected by tool posture, cutting conditions and path spacing. Wang et al. (2020) analyzed the surface topography of five-axis flank milling and controlled it, while Fu et al. (2023) assessed and classified the surface texture of the tool with the change of tool posture. Residual stress has attracted increasing attention, because it is the main concern of fatigue performance: Zhao et al. (2023) monitored the residual stress from the power consumption of the machine tool, and Wang et al. (2023) predicted the residual stress during the machining process by analyzing the cutting force parameters.

Therefore, the surface integrity is more comprehensive than the surface roughness and the surface roughness is not the only acceptance criterion of safety-critical parts. This is echoed by the multi-response studies on the outside of the five-axis corpus. The settings that produced minimum roughness (2000 rpm) did not match with the settings that produced minimum tool wear (1000 rpm) and GRA had to choose a compromise option (Mapindu et al., 2026b). In the same vein, the dilemmas between quality and productivity response were solved using Taguchi-GRA in the cases of fused deposition modelling (Mapindu et al., 2026c) and injection moulding (Mapindu et al., 2026a). For 5 axis planning, this implies that surface-integrity objectives must be explicitly addressed as additional responses and not taken for granted, as being a consequence of geometric accuracy.

5.6 Geometric error, measurement and compensation

The geometric relationship between machine axis, rotary tables and set up of workpiece and controller is the five axis accuracy. Liu et al. (2018) proposed a generalised inverse-kinematic model for geometric-error compensation and Xiang et al. (2018) and Fu et al. (2018) identified the individual error components and made a model of them. The sensitivity of each axis to the geometric error was assessed by using the theory of the product of exponentials (Fu et al., 2019). Errors that occurred from on machine measurement of the workpiece were compensated by Sato & Shirase (2018), which is a direct path from the machining results to the compensation. Li et al. (2019), He et al. (2024) optimized the point location and distribution of the measurement points for the measurement of volumetric error. This theme bridges the gap between planning and the actual machine, however, most compensation systems are set for one machine and have yet to be integrated with toolpath and orientation optimisation.

5.7 Digital twins and data-driven methods

Digital-twin approaches keep a virtual model of the machining process and incorporate process data. Zhao et al. (2022) optimized surface quality in five-axis manufacturing by tuning the machining parameters with a digital twin and Rudel et al. (2022) presented a cloud-based digital-twin solution that integrates coupled technology models for blisk milling. Based on these studies, it has been concluded that the digital twin can be used to connect planning and execution. Additionally, machine learning is being used in the planning process: Kukreja and Pande (2023) used machine learning to predict an optimal toolpath strategy in a 3D milling process; Chien, Sencer and Ward (2024) used a bidirectional long short-term memory network to predict five-axis cycle times in a milling process; Lu et al. (2025) introduced machine learning for energy-efficient flank-milling path generation. Chichell et al. (2026) combined toolpath and motion planning in an evolution-based optimiser.

A combination of maintenance and monitoring provides evidence of what these systems need for real. Based on the boiler's 13 years of data, a random-forest model was developed that found an optimal region for reliability, but testing showed that the difference between the model-optimised region and the actual region was too large to be resolved without integration with work-management routines (Senda et al., 2026). The multi-agent condition-monitoring system for an induction furnace was a combination of multiple programmable logic controllers (PLCs), Open Platform Communications (OPC) server and rule-based diagnostic agents that detected all the injected fault scenarios on a test bed, with in-situ validation still to be accomplished (Mashava et al., 2026). The same architecture, that is, sensors providing input to agents reasoning over rules or learned models, is directly applicable to closed loop multi-axis machining, where while cutting the spindle power and axis tracking error and vibration may update process models.

5.8 Energy and sustainability

Energy is added as an objective that interacts with all other objectives. In the past, Xu et al. (2016) designed toolpaths to minimize the amount of energy the machine consumes, Vishnu and Varghese (2020) used process-planning data to predict the amount of energy consumed by the machine and Lu et al. (2025) integrated energy into a reinforcement-learning path generator. Energy could not be derived from time or quality of product as in the case of honing, optimum energy was obtained at a different setting for surface roughness, power and total energy (Tayisepi et al., 2024), and in the wood planing, optimum energy was obtained at different settings for depth of cut and number of knives depending on the wood species used (Tayisepi et al., 2023). When the energy is included as a by-product of cycle time, it is not correct for a multi-axis machine that includes energy terms from the inclusion of rotary-axis motion and feedrate modulation.

6 DISCUSSION

6.1 Cross-study synthesis

Table 3 outlines the consensus and areas of disagreement in the studies. There are four main points that stand out from the conclusions. Firstly, the problem of geometry and machine motion must be considered simultaneously since orientating decisions are converted into rotary-axis motion that can have a significant impact on accuracy and time (Cripps et al., 2017; Huang, Lu and Zhu, 2018; Chen et al., 2025). Second, there are no universally best toolpath

strategies: iso-scallop toolpaths are more appropriate for surfaces where residual height is important; flank milling is more efficient for suitable ruled surfaces; trochoidal toolpaths control engagement in deep or constrained surfaces; barrel or custom tools expand the effective width of the tool, but are more complicated to orient. Third, an efficient path changes meaning, due to engagement changing with posture and this impacts on force and stability (Berglind, Plakhotnik and Ozturk, 2017; Li, Kilic and Altintas, 2020). Fourth, prediction and search can be enhanced by data-driven methods, but these are still not widely generalisable and have not been tested in industry.

Table 3: Areas of agreement and unresolved tension across the included studies

Research area	Agreement across studies	Important difference or tension
Tool orientation	Orientation affects accessibility, surface quality and axis motion	The preferred orientation depends on geometry, machine and objective weighting
Toolpath strategy	Different strategies suit different geometries and operation stages	No single strategy dominates accuracy, time and stability simultaneously
Feedrate and interpolation	Programmed feedrate is not identical to realised machine motion	Controller and machine architecture limit transferability
Process modelling	Engagement and posture affect cutting force and stability	Model fidelity competes with computational cost
Surface quality	Roughness and topography depend on tool posture and cutting conditions	Roughness alone does not capture residual stress or subsurface integrity
Intelligent optimisation	Data-driven methods can improve prediction or search efficiency	Generalisability and real-time validation remain limited

6.2 An integrated view of the multi-axis planning problem

Multi-axis planning is described as a coupled decision cycle in figure 5. CAD geometry determines the toolpath and orientation, which are translated into axis motion by machine kinematics and into the process physics by motion and engagement which in turn determines surface quality and integrity, which in turn determines time, energy and cost, which determine choice of strategy. All nodes can be linked via a digital twin or data feedback. The implication is that optimal toolpath must be qualified.

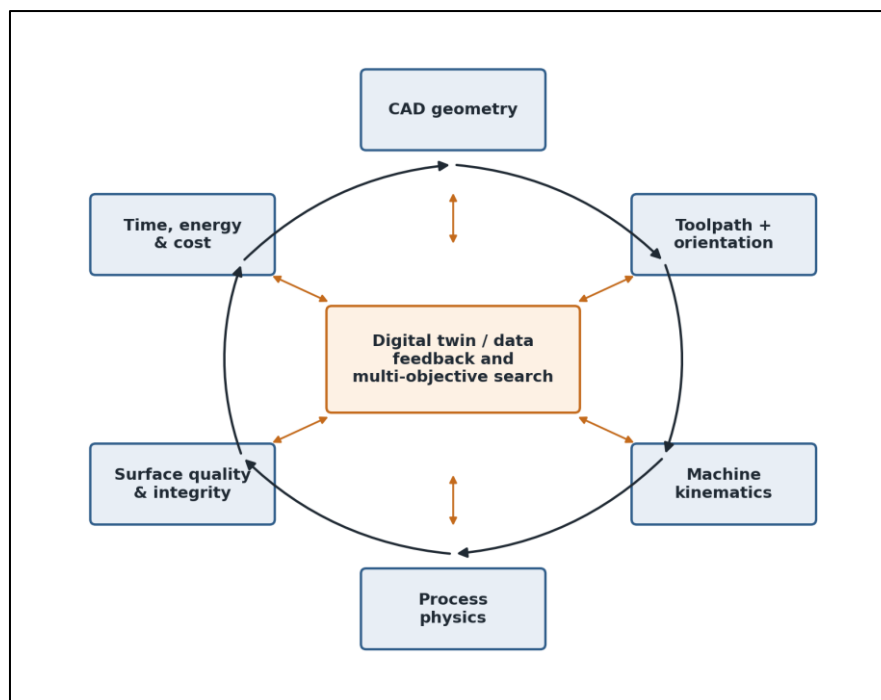


Figure 5: Coupled decision cycle in multi-axis CNC planning, with a digital-twin and multi-objective search layer connecting all decisions. Source: authors' own construction.

All of these can be beneficial: shorter paths may lead to more rotary axis, smoother orientation may lead to less axis acceleration but more path orientation changes, and faster feedrate may lead to less time, but more force and surface error. The literature thus justifies limited or Pareto like multiple objective formulation and not a single common objective.

Such formulations are practical, and are offered by the experimental optimisation literature. The sequence of using Taguchi designs with ANOVA to quantify the contribution of each parameter to each response and using GRA to convert conflicting responses into a single grade has been successfully applied in milling (Mapindu et al., 2026b), honing (Tayisepe et al., 2023), wood planning (Tayisepe et al., 2024), injection moulding (Mapindu et al., 2026a), additive manufacturing (Mapindu et al., 2026c), in weld parameter optimization (Mapindu et al., 2026d) and in edible-oil bleaching, regression-based predictive modelling from designed experiments has guided the process design (Jansen et al., 2025). Where machine time and material are limited, these methods are satisfactory in experimental runs, but do not allow separation of interaction effects as does a saturated orthogonal array. To make such generalisations, designs that resolve interactions, or physics-based surrogate models, are needed, as the orientation of the tool has a strong interaction with the feed and engagement in five axis machining.

Table 4: Transferable evidence from the authors' research group and its relevance to multi-axis machining

Study	Process context or	Method	Key finding	Relevance to multi-axis CNC
Mapindu et al. (2026b)	CNC milling of mild steel	Taguchi L9, ANOVA, GRA	Feed rate explained 82.81% of roughness variation; spindle speed 63.40% of flank-wear variation; GRA compromise at 1000 rpm, 0.10 mm/rev, 0.5 mm	Quality and tool-life optima conflict and need explicit multi-response selection
Tayisepe et al. (2024)	Honing of cast-iron cylinder liners	Taguchi L16, ANOVA, S/N ratio	Different optima for roughness, power and energy; confirmation within 10%	Energy must be an explicit planning objective
Tayisepe et al. (2023)	Planing of teak, saligna and pine	Taguchi L9, ANOVA, regression	Energy- and quality-optimal settings change with species	Optimal settings are material-specific
Mapindu et al. (2026a)	Injection moulding	Taguchi L9, ANOVA, GRA	Injection pressure dominated shrinkage (67.25%) and warpage (60.55%)	Mould cavities are a key multi-axis application; multi-response method
Mapindu et al. (2026c)	Fused deposition modelling of PLA	Taguchi L9, ANOVA, GRA	Infill density dominated strength (68.99%); layer height dominated print time (51.77%)	Quality-time trade-off; hybrid additive-subtractive routes
Jansen et al. (2025)	Vacuum bleaching of edible oil	Taguchi L12, regression prediction	Oil temperature significant for free fatty acids; regression models guided system design	Designed experiments as low-cost surrogate models
Senda et al. (2026)	Boiler maintenance planning	Random forest on 13-year dataset	Optimal reliability region of about 66%; gap between model and practice	Data-driven models must be embedded in operating routines
Mashava et al. (2026)	Induction-furnace condition monitoring	PLC-OPC multi-agent system with rule-based diagnosis	All five injected faults detected; no false positives in 72 h	Reference architecture for closed-loop monitoring
Muhla et al. (2025a)	Industry adoption 4.0	Industry survey and framework	Lack of strategy, cost and weak policy support were the main barriers	Adoption context for digital CAM
Muhla et al. (2023)	Manufacturing agility	Industry survey and literature synthesis	NC tools and rapid-prototyping equipment in short supply	Multi-axis capability as an agility enabler

Muhla et al. (2025b)	Workplace safety	Mixed-methods survey (87 responses)	Cognitive overload, fatigue and training gaps contribute to accidents	Operator workload and collision risk
Ndiyamba et al. (2026)	Engineering education	Framework development	Regional learning-factory network proposed	Skills pathway for multi-axis CAM
Muyambo et al. (2025)	Vertical-axis wind turbine	Analytical design, CFD, prototype	Prototype produced up to 45 W from vehicle slipstream	Aerofoil blades as free-form parts
Tandi et al. (2025)	Passive exoskeleton	CAD, FEA, topology optimisation	2.6 kg device at about US\$387	Topology-optimised parts require complex-geometry machining
Munotyaani et al. (2026)	Smart wheelchair	Systematic design, microcontroller prototype	Obstacle detection within 50 cm at 60–70% lower cost	Low-cost, locally maintainable engineering

6.3 Methodological differences across the literature

Studies vary in their method of substantiation. Many toolpath and orientation papers are verified by a simulation of scallop heights, gouging or axis motion, and only a single machining test is conducted on one geometry. In process-physics papers, typically the force or stability agreement is reported for only a subset of engagements. Geometric-error and measurement papers are checked on one machine configuration. There are not many studies that report both energy and residual stress, and even less than that report energy and residual stress on more than one machine architecture. There is no common geometry and data set that can be used to compare methods directly, and several studies report better results than a baseline CAM strategy, the settings of which are not well-documented. The differences reduce the credibility of the cross-study conclusions and highlight the need for standardising validators.

6.4 Implications for industry and for developing industrial economies

In terms of the practitioners, the synthesis allows them to choose toolpath strategy by region and operation instead of the whole part; verify the candidate paths in the machine joint space including singularities and rotary axis reversals before cutting; and takes energy and surface integrity as a clear acceptance criterion for critical parts. The engineering implications and remaining limitations are listed in table 5.

Table 5: Engineering implications drawn from the cross-study synthesis

Observed pattern	Engineering implication	Remaining limitation
Orientation changes affect both geometry and rotary-axis motion	Plan posture together with the path and verify in joint space	Machine-specific inverse kinematics is still difficult to generalise
Toolpath families perform differently on different geometries	Select strategies by region and operation	Automated strategy selection needs reliable geometry descriptors
Cutting force depends on engagement and posture	Include process models where stability matters	High-fidelity models can be computationally expensive
Digital twins can connect planning and execution	Use process data to update predictions	Closed-loop industrial validation remains limited
ML and RL can search large decision spaces	Use data-driven optimisation for complex objectives	Training data and safe constraints are major barriers
Energy can be included in planning	Treat energy together with time and quality	Energy models remain machine- and process-dependent

Barriers in developing industrial economies are as much organisational as technical. In the Zimbabwean context, the key barriers for implementing Industry 4.0 technologies faced by manufacturers were the lack of an implementation strategy, high implementation costs and low levels of policy implementation (Muhla et al., 2025a), while even manufacturers aiming to achieve agility were lacking in the availability of rapid-prototyping and numerically controlled equipment (Muhla et al., 2023). Cognitive overload, fatigue and insufficient training were found to be factors that

contribute to workplace accidents in the same industry through human-factors research (Muhla et al., 2025b), which is applicable as multi-axis machining increases the cognitive load of the programmer and operators while also elevating collision risk. Learning factories have been suggested as a regional means for fostering the development of these skills (Ndiyamba et al., 2026). Low-cost, locally maintainable engineering solutions have been demonstrated in assistive technology (Munotyaani et al., 2026), renewable-energy harvesting (Muyambo et al., 2025) and ergonomic devices (Tandi et al., 2025), and a comparable approach, combining open CAM verification tools, shared machine models and staged pilot implementation, is likely to be more effective than direct transfer of high-end digital-twin platforms.

7 RESEARCH GAPS AND FUTURE DIRECTIONS

Table 6 summarises the research agenda. The gaps below follow from the synthesis in Sections 5 and 6.

7.1 Machine- and controller-aware CAM

The largest gap is the separation between CAM geometry and actual machine behaviour. Path planning should include inverse kinematics, controller interpolation and acceleration and jerk limits during optimisation rather than check them after path generation (Chen et al., 2025; Wu et al., 2025). Portable machine models that can be calibrated for different architectures would make research results more transferable.

7.2 Generalisable artificial intelligence and reinforcement learning

Machine learning and reinforcement learning are used increasingly because the multi-axis decision space is difficult to search with conventional optimisation alone (Kukreja and Pande, 2023; Lu et al., 2025). The unresolved issue is generalisation: a model trained on one machine or component may not behave similarly on another. Transfer learning, physics-informed learning, domain adaptation and explicit constraint handling are promising directions, and the maintenance literature shows that model outputs must be embedded in operating routines before their predicted benefit is realised (Senda et al., 2026).

7.3 Surface integrity beyond roughness

Future work should give more weight to residual stress, subsurface damage, microstructural effects and fatigue-relevant surface condition, particularly for aerospace and other safety-critical components (Zhao et al., 2023; Wang et al., 2023). Toolpath optimisation should consider the functional state of the surface rather than treat roughness as the final quality measure.

7.4 Closed-loop digital twins and in-process monitoring

Current digital twins mainly adjust parameters within a fixed toolpath (Zhao et al., 2022; Rudel et al., 2022). Closing the loop at the level of path and orientation requires in-process sensing, fast surrogate models and decision logic that can act within the controller cycle. Agent-based monitoring architectures that integrate PLC data, OPC connectivity and rule-based or learned diagnostics (Mashava et al., 2026) offer a tested pattern for this integration.

7.5 Energy-aware multi-objective planning

Energy should be optimised jointly with time and quality using machine-specific models that include rotary-axis and spindle power. The divergence between energy-optimal and quality-optimal settings observed in honing and wood machining (Tayisepi et al., 2023; Tayisepi et al., 2024) suggests that similar trade-offs will appear in five-axis cutting and should be quantified.

7.6 Benchmarks, datasets and capability building

The field needs shared benchmark geometries, open datasets of axis motion, force and surface data, and common reporting of baseline CAM settings. For developing economies, research should also study the adoption pathway: skills, affordable verification tools and learning-factory environments in which multi-axis CAM can be taught and tested (Ndiyamba et al., 2026; Muhla et al., 2025a).

Table 6: Research agenda for multi-axis CNC toolpath planning

Gap	Proposed direction	Example research question
CAM separated from machine behaviour	Machine- and controller-aware optimisation	How much cycle time and contour error are recovered when jerk and singularity constraints are optimised with the path?
Poor generalisation of learned models	Transfer, physics-informed and constrained learning	Can a policy trained on one table–table machine be adapted to a head–head machine with limited data?
Roughness used as sole quality measure	Surface-integrity objectives in planning	How do orientation and engagement fields affect residual stress along a blade?
Open-loop digital twins	In-process sensing with agent-based decision logic	Which sensor set and update rate are sufficient to adjust orientation during cutting?
Energy treated as secondary	Machine-specific energy models in multi-objective search	What is the Pareto front between energy, time and scallop height for a representative impeller?
No shared benchmarks; weak capability in developing economies	Open datasets, benchmark parts and learning-factory testbeds	Which low-cost verification workflow gives acceptable collision and error risk for SMEs?

8 CONCLUSIONS

A review of the literature was conducted and the results of 51 primary studies and two previous reviews, published from 2016 to 2026, were synthesized on the topic of multi-axis CNC machining of complex geometries. Multi-axis planning has evolved from toolpath generation based on geometry toward integrated tool position and orientation planning, machine motion and process behavior. To answer RQ1, the literature provides iso-scallop, partition-based, flank, trochoidal, barrel and non-spherical tool strategies and orientation methods based on rotation-minimising frames, tractrix fields, feasible regions and convex corridors.

They are not suitable for all conditions and geometries. For the answer to RQ2, it is difficult to distinguish geometric accuracy from machine kinematics and process stability. When tool orientation changes, singularities, rotary-axis motion, feedrate, cutting force, surface integrity change, and the toolpaths need to be tested in the machine domain and, if suitable, using process models. Optimisation is now no longer limited to geometric algorithms and designed experiments, but instead involves machine learning, digital twins and reinforcement learning, as a response to RQ3. These methods are potential for cycle-time, energy, surface-quality and strategy-selection problems; however, the physical constraints and machine-specific validation is needed.

For RQ4, the key gaps identified are machine- and controller-aware CAM, generalisable learning methods, learning about surface integrity beyond roughness, closed-loop digital twin, energy-aware multi-objective planning, and common benchmarks. Capability building and a gradual, low-cost adoption are just as critical to industrial economies as algorithmic innovations are. There are limitations to the review. It used primarily one bibliographic source, with a small amount of citation chasing, contained only English language literature, and coded each study to a single main theme, many of which were multi-themed. For future updates: Expand search to Scopus and Web of Science and perform double independent screening.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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