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# Projection of population growth of Wukari Local Government, Taraba State, using three mathematical population models

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## Abstract

Government usually spend huge amount of money when embarking on census in other to know the population of the people living in a community. Also, despite the huge amount of money government invest on census; some of the areas are not captured. This study is aimed at the use of Mathematical models to project the future population of Wukari Local Government Area which will determine how government can embark on conducting census at a reduced or minimized cost. Data were collected form the Statistical Year Book of Taraba State. The data spanned from 2006 to 2016. The following models; Malthusian model; Arithmetic and Geometric model were used in the projection of the population of Wukari Local Government Area from 2016 to 2040. The results show that Malthusian model has minimum error compared to the Arithmetic and Geometric models.

**Keywords:** Population; Census; Mathematical model; Government; Projection

## 1. Introduction

The knowledge of population and the rate of growth of the population of any community is very expedient for any effective economic and developmental planning of that community. This is so, because human wants are infinite and the resources available to satisfy these needs are limited. Therefore, when the population of a community is known, then the government as well as non-governmental organizations will know how to plan and allocate the limited resources to the community.

The study of population is very important, since it tends to be the determining factor or tool for the distribution of the available limited resources in the community.

Population can be defined as the whole number of people or inhabitants in a country or region. It is the total of individuals occupying an area or making up a whole.

The population growth rate, is the rate at which the number of individuals in a population increases in a given time period, expressed as a fraction of the initial population. Specifically, population growth rate refers to the change in population over a unit time period, often expressed as a percentage of the number of individuals in a population at the beginning of that period.

The positive growth rate indicates that the population is increasing, while a negative growth indicates that the population is decreasing. A growth ratio of zero indicates that there were the same number of individual at beginning

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and end of the period a growth rate may not be zero even when there are no significant changes in the birth rates, death rates, immigration rates, and age distribution between two times.

A related measure is the net reproduction rate. In the absence of immigration, a net reproduction rate of more than 1 indicates that the population of females is increasing, while a net reproduction rate less than one (sub-replacement fertility) indicates that the population of females is decreasing.

Many people have done a lot of work on population but to the best of my knowledge, no one has projected the population growth of Wukari. For instance, [20], stated that the world population is growing. By the end of the century, it is expected to increase by 3.7 billion people, rising from 7.2 billion in mid-2013 to 10.9 billion by 2100[19]. The distribution of future population growth by countries, regions and major areas is readily available from population projections prepared by the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat [20].

In the research work titled "Aging and Real Interest Rate in the last and Next 50 years carried out by [16], they observed that Population aging, along with a secular decline in real interest rates, is an empirical regularity observed in developed countries over the last few decades. Under the premise that population aging will deepen further in coming years.

[14], stated that fertility provides a positive contribution to population growth if fertility is above replacement and a negative contribution to population growth if fertility is below replacement. The concept of replacement fertility is important in population projections because maintaining fertility at replacement level in the long run leads to a stationary population and stabilization of population growth.

[18], started his career as a physicist but then did his post-doctoral work in applied mathematics. He became very interesting in the mathematical explanations of what enables competing species to coexist and in the mathematics behind population growth. Most of the findings on the basic Arithmetic population growth equation on the value of r.

In the study conducted by [12], as reported by [4], the link between population, per capita growth and poverty in Uganda was examined. The research was conducted using both cross-section data and panel data. The results of the estimates show that population growth has a positive impact on overall economic growth.

[6], point out the possibility that population aging, combined with the facts of other factors, and could bring real interest rates down to a value below zero for a prolonged period.

According to, [11], they observed that, Ecologists are often interested in how populations, communities, and ecosystems change in space and time.

[2] argued that increasing population is very important in the development process of LDCs because labour or human capital is a major component in the production process. They however argue that the population is a blessing if a large part of such population constitutes well-trained and informed human capital [1]; [9].

Such training that is required involves investment in education and health, which is an effective way a nation's population can be transformed into a strong and efficient labour stock with high potentials for economic productivity. Population projections, like official statistics in general, are intended to serve the information systems of democracies for better decision making. Independence and impartiality of population projections are preconditions for fulfilling this demanding role [21].

Calibration is indeed an important tool for projection makers in developing plausible estimates of the uncertainty associated with population projections. For example, observed values are expected to be contained in an 80 per cent prediction interval, on average 80 per cent of the time [15].

Calibration can be achieved by using sequences of retrospective forecasts, where a past period is forecasted using data available at the beginning of the forecast. By measuring how a forecast methodology performs in past contexts, retrospective forecasts also provide an indication of how it is likely to perform in the future [13].

Results from cognitive research show that calibration tends to improve trust in forecasts [15].

[10] brought the idea of estimating world population at 7.4 billion and project it to increase to 8.5 billion in 14 years and 9.9 billion in the next 26 years.

The current world population of 7.2 billion is projected to increase by 1 billion over the next 12 years and reach 9.6 billion by 2050, according to a United Nations report launched today, which points out that growth will be mainly in developing countries, with more than half in Africa. Although population growth has slowed for the world as a whole, this report reminds us that some developing countries, especially in Africa, are still growing rapidly [22].

[3] says that the world's population is now odds-on to swell ever-higher for the rest of the century, posing grave challenges for food supplies, healthcare and social cohesion. A ground-breaking analysis released on Thursday shows there is a 70% chance that the number of people on the planet will rise continuously from 7bn today to 11bn in 2100."

## 2. Material and methods

The mathematical modeling methodology enables us to transform real world problems to mathematical model based on certain assumptions about the real world concerned, and then, solve and interpret the mathematical solution.

In this work, we used mathematical models to predict the population of Wukari Local Government Area over a period of Thirty-four (34) years. The models that were used are; Malthusian growth model, Arithmetic model and geometric growth model.

### 1.1. Malthusian (exponential) Growth model

The first mathematical model, referred to as the classical Malthusian scheme for population growth, is based on the work by Thomas, (1766 - 1834). In the principle of population essay that he published in 1798, Malthus explained in fundamental and brilliantly simple terms his theories of human population growth and the connection between over-population and misery. One of the fundamental concepts that he brought up is that of unlimited population growth. The mathematical model based on this idea is that population size for one generation depends on the size of the previous generation, and it is a multiple. Assumptions of the Malthusian (exponential) model.

- The birth rate and death rate are considered to be the same for all the intervals.
- The environment inhabited by the population under study is considered to be close in terms of migration.
- That the birth and death are proportional to the population size and the time interval.
- Reproduction is considered to be a continuous process by all the individuals of the population.

### 1.2. The Model Equation for the Malthusian (Exponential) Model

From the assumption mentioned above; it is assumed that both the birth and deaths are proportional to the population size and the time interval.

That is;

$$Birth = aNdt$$

$$Death = bNdt$$

Where b and a are constant

Thus the increase, say $dN$ in the total interval $dt$ is given by $dN = aNdt - bNdt = rNdt$

Where r=a-b

Hence;

$$dN = rNdt \qquad (1)$$

Dividing equation (1) by $dt$ and taking the limit as $dt \to 0$, lead to the differential equation.

$$\frac{dN}{dt} = rN \qquad (2)$$

Multiply equation (2)by $y\frac{dt}{N}$, we obtain.

$$\frac{dN}{N} = rdt \quad (3)$$

And integrating both sides we have

$$\text{Log}\, N = rt + A \quad (4)$$

And if at t=0, $N = N_0$, We have $\log N_0 = A\ thus$

$$N_t = N_0 e^{rt} \quad (5)$$

Where

r =Growth rate

$N_{(t)}$=The number of people at the arbitrary time t

$N_0$= Total number of people at the initial time

t = Time of growth

### 1.3. Arithmetic Model

Arithmetic model which can simply be referred to as the mean or average of the data set as a statistical sample.

*1.3.1. Assumption (for the Arithmetic model).*

For the Arithmetic model to be realistic, the following assumptions must be considered.

- The birth rate and death rate are considered to be the same for all intervals.
- The environment inhabited by the population under study is considered to be close in terms of migration.
- The population is only considered with the people living in that environment.
- Reproduction is considered to be a continuous process by all individuals of the population.

*1.3.2. Arithmetic equation*

$$P_0(1 + rt) \quad (6)$$

Where r= growth rate.

$p_o$ = Total number of people at the initial time.

t = time of growth.

b (t) = death rate at time t.

d (t) = death rate at t.

p (t)= the number of people at the arbitrary time t.

*1.3.3. SOLUTION OF THE EQUATION*

$$Pt = P_0(1 + rt) \quad (7)$$

By opening the bracket, we therefore have

$$Pt = P_0X1 + P_0Xrt \quad (8)$$

$Pt = P_0 + rtP_0$ (9)

As our equation we collect like terms

$Pt + P_0 = rtP_0$ (10)

Divide both side by $tP_0$ we have

$\frac{Pt-P_0}{tP_0} = \frac{rtP_0}{tP_0}$

$= \frac{Pt-P_0}{tP_0}$ (11)

### 1.4. Geometric model

Geometric growth refers to the situation where successive change in a population differs by a constant ratio (as distinct amount).

### 1.5. Geometric assumption model

$P_{t+n}=P_t(1+r)^n$ (12)

Taking the $n^{th}$ root of both sides

$\sqrt[n]{P_{t+n}} = \sqrt[n]{P_t}\sqrt{(1+r)^n}$ (13)

$\sqrt[n]{P_{t+n}} = \sqrt[n]{P_t}\,(1+r)$

$P_{t+n} = P_t + rP_t$ (14)

Collecting the like terms

$P_{t+n} - P_t = rP_t$ (15)

Taking log and dividing both sides by Pt

$\frac{P_{t+n}-P_t}{P_t} = \frac{rP_t}{P_t}$ (16)

$n = \frac{\left(\frac{log\, P_{t+n}}{log\, P_t}\right)}{log(1+r)}$ (17)

$= \frac{log\left(\frac{P_{t+n}}{P_t}\right)}{log(1+r)}$

Finally, we have

$= \left(\frac{P_{t+n}}{P_t}\right)^{\frac{1}{n}} - 1$ (18)

## 3. Results and discussion

The three models stated above were used to analyze the data in table 1. Comparison of the projected results from Malthusian model, Arithmetic model, and Geometric model were considered. However, the model with minimum error is chosen for projection of the population of Wukari Local Government for the period of 34 years. The results from the three models are as presented in the table below.

**Table 1** Malthusian Model, Arithmetic model and Geometric model population forecast for Wukari Local Government

| Years | Malthusian Model (Pop) | Malthusian Errors | Arithmetic Model (Pop) | Arithmetic Errors | Geometric Model (Pop) | Geometric Errors |
|---|---|---|---|---|---|---|
| 2016 | 318,400 | 564.269 | 318,400 | 564.269 | 318,400 | 564.269 |
| 2018 | 322,113 | 567.549 | 457404.7 | 676.317 | 415944.8 | 644.937 |
| 2020 | 325,869.3 | 570.849 | 657094.9 | 810.613 | 543373.2 | 737.138 |
| 2022 | 329,669.4 | 574.116 | 943964.4 | 971.578 | 709840.6 | 842.520 |
| 2024 | 333,513.9 | 577.506 | 1356073 | 1164.505 | 927306.8 | 962.967 |
| 2026 | 337403.9 | 580.864 | 1948097 | 1395.742 | 1211396 | 1100.634 |
| 2028 | 341,337.7 | 584.241 | 2498583 | 1580.690 | 1582518 | 1257.981 |
| 2030 | 345,318.2 | 587.637 | 4020367 | 6343.246 | 2067337 | 1437.823 |
| 2032 | 349,345.2 | 591.054 | 5775548 | 2403.234 | 2700684 | 1643.375 |
| 2034 | 353,419 | 594.490 | 3296993 | 1818.762 | 3528064 | 1878.314 |
| 2036 | 357,540.4 | 597.946 | 11919231 | 3452.423 | 4608918 | 2146.639 |
| 2038 | 361,709.9 | 601.423 | 17122839 | 4137.975 | 6020903 | 2453.752 |
| 2040 | 365,927.9 | 604.919 | 24598198 | 4959.657 | 7865461 | 2804.542 |

## 4. Discussion

From table 1 of our analysis we observed that the Malthusian mathematical model has the minimum error which means that it is the best mathematical model among the three models.

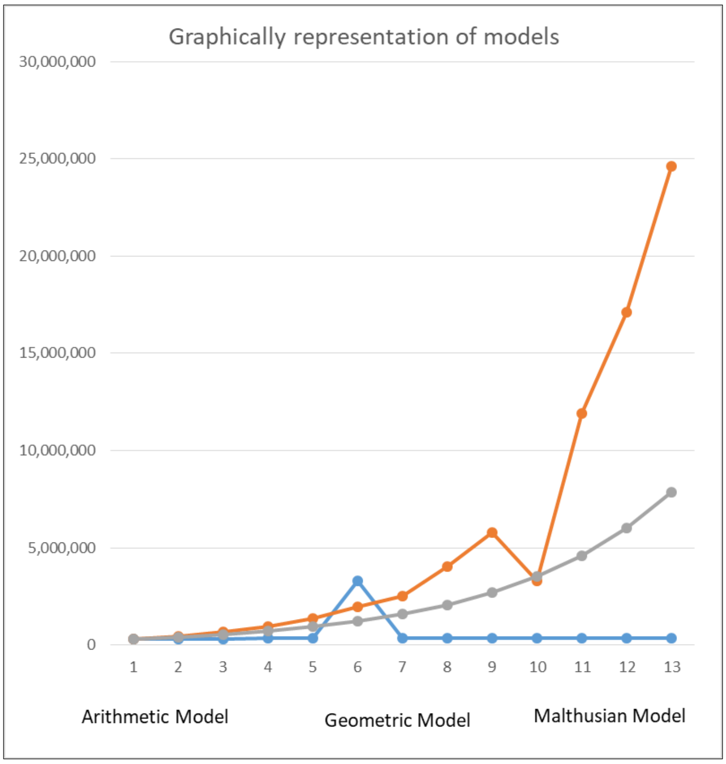


**Figure 1** Graph of comparison between Malthusian Model, Arithmetic Model and Geometric Model

| | |
|---|---|
| Malthusian | Series 3 |
| Arithmetic | Series 2 |
| Geometric | Series 1 |

From figure 1 above, we observed that in the X – axis the Arithmetic model has a constant growth from 0 to 5 with population growth of approximately zero population it then increased to 4,000000 between the period of 5 to 7 and move back to constant growth of zero population.

In geometric model, we observed that the graph is meandering and later moved with a linear population growth of 25,000000.

In Malthusian model, there is a linear increase in the population growth and it performs better than the other two models

## 5. Conclusion

The outcome of the comparison analysis between the three models for this study showed that the Malthusian mathematical model performed better in estimating the population of Wukari Local Government people. This is because the growth rate from the Malthusian model was a steady and logistic growth as shown in figure 1 and table1 above.

## Compliance with ethical standards

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### *Disclosure of conflict of interest*

The Authors declare that there is no conflict of interest.

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