Open Access Research Journal of
Science and Technology
Journals home page: https://oarjst.com/
ISSN: 2782-9960 (Online)

OARJ OPEN ACCESS RESEARCH JOURNALS

(Review Article)

Check for updates

The six dimensions of SU (3)

Gudrun Kalmbach H.E. *

Mint PF 1533, D-86818 Bad Woerishofen, Germany.

Open Access Research Journal of Science and Technology, 2022, 05(01), 001–006

Publication history: Received on 16 February 2021; revised on 21 March 2021; accepted on 24 March 2021

Article DOI: https://doi.org/10.53022/oarjst.2022.5.1.0023

Abstract

The QCD theory uses eight generating GellMann matrices for its SU(3) symmetry presenting the strong interaction. Instead of attributing the basic six color charges in QCD only to gluons and quarks, they are treated as an independent energy cc which has its six values from the sequence of complex cross ratios. The space for them is a complex C^3 with an energy plane added to spacetime. In octonian coordinates, two octonian dimensions are missing. Octonian models are presented where the color charges act independent of QCD. The catastrophe, models and field quantum used are the parabolic umbilic, the 6 roll mill, the hedgehog and rgb-gravitons, generating barycentrical coordinates where Higgs can set mass as weight at a barycenter. The conic color charge whirls rgb red-green-blue are in superposition the neutral color charge of all baryons and nucleons.

Keywords: SU; Six dimensions; QCD; GellMann matrices

1. Introduction

The concept of color charge was introduced because uud or ddu quarks in a nucleon needed an additional quantum number. The Pauli exclusion principle requires this.

In 1988 the author suggested that the new charges are independent of quarks. In the strong interaction SI theory only 8 gluons as superpositions of 2,4 or 6 color charges are used. This has to be extended.

Color charges can have the observed neutral superposition of all baryons or nucleons red-green-blue in rgb-gravitons. This field quantum belongs to gravity, not SU(3). Therefore color charges are treated as an independent energy from quarks quantum number.

For gravity rgb-gravitons serve in the same way to mass as a scalar charge, similar to magnetic momentum which serves for electrical charge. They are a Gleason operator frame GF consisting of three pairwise orthogonal base vectors (r,g,b), similar to the spin triple for space as s = (sx,sy,sz). For spin the symmetry of Pauli matrices σj and the quaternionic number system is accepted from physics. The symmetry SU(2) has σj as generators and the weak interaction uses them for the three weak bosons as field quantums. If the identity operator and the matrices are signed, there is a discrete group of order 8 for SU(2). It has cyclic subgroups of order 4 which can be interpreted as {1,i,-1,-i}, introducing complex numbers. Spin coordinates for space are extended by time to a

4-dimensional Hilbert space. It has a Euclidean metric for space, but for time the diagonalized metric is negative in [1,1,1,-1] for the Minkowski metric. It can be argued that time is used imaginary and scaled with the constant speed of light c as ict. Many GF have dimensional extensions like this.

* Corresponding author: Gudrun Kalmbach H.E
Mint PF 1533, D-86818 Bad Woerishofen, Germany.

Th three whirls r,g,b in superposition can be similar interpreted as magnetic field quantum whirls for the electromagnetic force EM where the EM charge has as a GF in octonian coordinates (e1,e4,e5) for electrical charge as weight at the first coordinate, magnetic energy at the second and induction B as cross product and rotational momentum at the third coordinate. If octonian coordinates are listed by their indices 0,1,2,…,7, the rgb-graviton GF is 126, spin 123 and EM magnetic momentum 145.

For gravity, mass replaces the EM charge. A Higgs boson sets a mass scalar for a system at a barycenter as intersection of barycentrical coordinates. In contrary spin sets a center for space 123 coordinates of a system and the EM coordinatized space 145 can be observed as the cross product for the definition of B. The integration/differentiation combines B with a magnetic field strength Φ in B = dΦ/dA where A is the area inside a flat electrical currents loop which is transversal crossed by the field Φ (figure 1).

2. Gravitons with color charges

The color charge space is different. The rgb-gravitons are not responsible for space coordinates xyz of spin, replacing them. They generate barycentrical coordinates in a quark triangle. The difference is that for quaternions the transformation matrices used are Euler angles. In rotating a single wheel, they generate one of the x, y, or z axis. Computations with Euler angles give then the Pauli matrix multiplication of quaternions (figure 1). The orthogonal base vector triple of space for the spin GF uses the real cross product for the quaternionic, non-commutative multiplication. This GF has as weights length units as meter for measuring distances with the Euclidean metric.

In contrary, the orthogonal base vector triple for rgb-gravitons GF has as weights attached the color charges r,g,b. They arise as Fibonacci like sequence from the charateristic polynomial z3 -1with suitable initial values for the sequence and solutions (p1n), {1, p1, p2} as third roots of unity. This replaces the {1, i,-1,-i} sequence for quaternions where the characteristic polynomial z4 -1 is used for getting this Fibonacci like sequence as solution of a difference equation. In both cases, difference equations with suitable initial values replace differential equations. The characteristic polynomials can be used for differential equations, but they generate not these two finite sequences. The Pauli case is for 90 degree rotations, spin coordinates and three Euler angles, the rgb-graviton case for 120 degree rotations and three barycentrical coordinates as axes of three conic color charge whirls r,g,b.

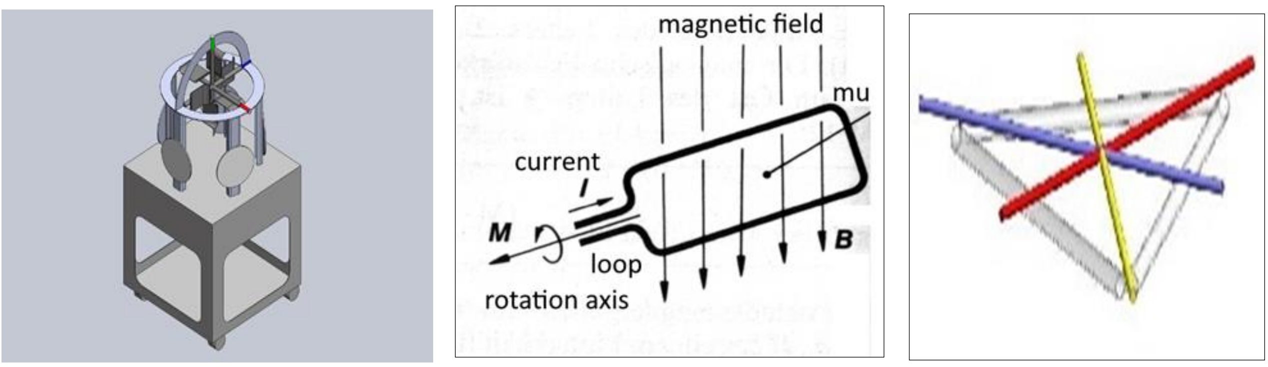

Figure 1 Wheel for 123 spin and space coordinates, middle 145 EM loop, at right barycentrical coordinates 126 rgb in a nucleons quark triangle

The third roots of unity are presented for 126 in the symmetry D3 of the quark triangle as a rotation of order 3, α,α^2, id. These are flat rotations, not spacial reflections like the Pauli matrices. The three

D3 reflections are for the rgb axes of conic rotations. The dynamics is described as the SI rotor, a representation of the D3 symmetry [2]. D3 has as degenerate orbits {p1,p2}, {1,-2,-1/2} and {0,-1,∞}.

The last orbit is used for generating the six complex cross ratios for gravity which replace the EM Pauli matrices. In books for complex variables, mostly -1 is replaced by +1. But this is not the irreducible representation for the SI rotor since α is missing. It has as 2x2-matrix its first row (0 1) and (-1 -1) as second row. The (first) Pauli matrix with first row (0 1) and second row (1 0) is a D3 reflection. The degenerate basic spin lengths orbit is used by rgb-gravitons for a pendulum contraction/expansion of the nucleon triangle during the SI rotors dynamics [2].

3. Coordinate space C^3

For GR with mass as charge and rgb-gravitons as field quantums the complex spacetime (z1,z2) of physics is extended by the complex cross product z3 = z1xz2 to a subspace of octonians by adding an energy plane z3 = (m,f) for a mass m coordinate with measure kg and a frequency f coordinate with 3 measure Hz. The line $mc^2 = E = hf$, E energy, belongs to this plane. The complex extention is used by SI for reduced coordinates (z1,z2 ,z3) (in octonian coordinates 123456) of its 8-dimensional

symmetry SU(3). The 8 SU(3) GellMann matrices are projection maps with a row and column of coordinates 0 added to Pauli matrices. In the former coordinates notation, their first coordinatized rows are then (z1 z2 0), (z1 0 z3) and (0 z2 z3). The first matrix is projected down by the rgb-gravitons down to spacetime and the weak interactions WI Hopf sphere S^3. It is a factor of the SU(3) geometry S^3xS5 . The second matrix is as real 4-dimensional space (z,ict,m,f) 3456 and the third matrix is (x,iy,m,f) 1256 where spacetime coordinates (x,y,z,ict) are included in the new lists. The GF 356 in 3456 is for the SI rotor with time 4 added for its discrete dynamics. The GF 126 in 1256 is for the gravitational field with rgb-gravitons as field quantums. It has as potential function Gm/r, the gravity fields strength, r radius, G gravitational constant. A common 5-dimensional potential field POT with EM is 12456 [7].

4. Roll mill, hedgehog and driving forces POT, SI, WI

In [8] the 6-dimensional complex space (z1,z2 ,z3), 123456 in octonians, is used for the color charges 6 roll mill, an elliptic umbilic catastrophe [5]. It is also used for the hedgehog of a deuteron atomic kernels with two nucleons, a proton and a neutron, having six quarks uud, ddu together. The 6 roll mill can be interpreted as a quarkgluon flow about the six r,g,b and their duals rolls. The mill is driven by three motors POT, SI and WI where the gluon exchange between quarks in the nucleons quark triangle is with SI on the rolls g and its dual c(g), the WI force adds weak bosons for an isospin exchange in a WI rotor between paired ud quarks on a coordinate line x,y or z on the rolls b, c(b) and gravity uses as POT motor the rolls r, c(r) for mass and its potential. POT and SI have the same speed and the WI speed is different.

The hedgehog model is not related to this catastrophe potential. It has a SI fiber bundle as geometry, similar to the SU(2) Hopf fiber bundle for EM. The space S5 of the SU(3) geometry is mapped to CP^2 with fiber S1 , a complex 2-dimensional inner spacetime for deuteron and atomic kernels AK with a bounding Riemannian sphere S^2. The symmetry of Moebius transformations MT is added to the symmetries of the standard model U(1)xSU(2)xSU(3), but not as a product. The cross ratios are the invariants of MT. The sphere S^2 occurs in three copies which are Heegard split at their equator into two hemispheres, interpreted as six color charge cones. They are paired according to the three Heisenberg uncertainties HU u,c(u), u = r,g,b. Along the caps axes are central energy normal vectors for the energy exchange AK with its environment, They can turn up/down like spin vectors for an energy output or input. As energies vectors theay are in octonian notation 1 EM(pot) red r for the electrical potential, 2 E(heat) green g for temperature, phonons exchange, 3 E(rot) rotational energy/angular momentum/angular speed, 4 E(magn) magnetic energy with magnetic field quantums, 5 E(pot) mass in kg measured, 6 E(kin) kinetic energy and momentum.

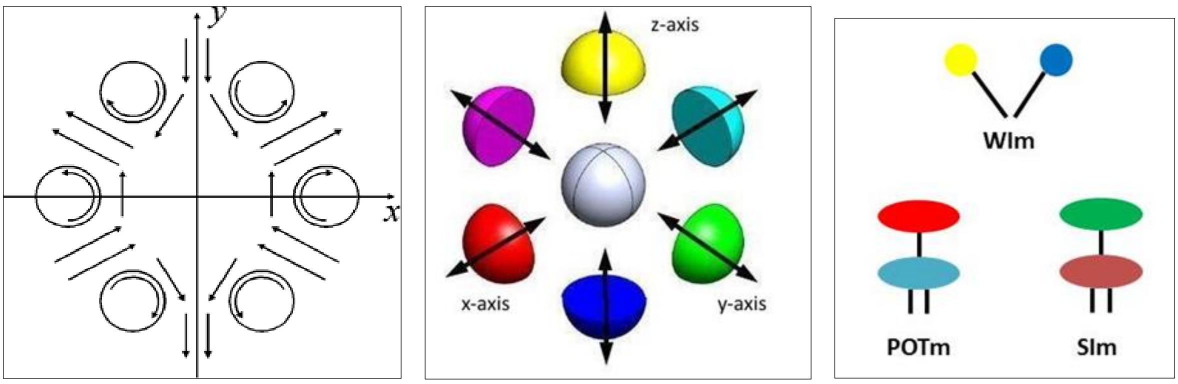

Figure 2 6 Roll mill, hedgehog, 3 motors with color charge rolls x, c(x), x=r for POT, x = g for SI,

x = b for WI

5. Parabolic Umbilic

For the thresholds as a kind of valve for the up/down turns of the vectors can act a catastrophe. It allows spontaneous, sudden changes, jumps through its potential function. Beside one or two variables the seven basic catastrophes have parameters. Counting them together, the parabolic umbilic has 6 dimensions and can serve for this purpose.

The parabolic umbilic has many cusps which allow jumps in potentials from one level to another.

At an umbilic point, two cones are glued together for a Minkowski double cone (at right from the cusp in figure 3). Somewhere in the parabolic umbilic occur as subgeometries the elliptic, hyperbolic umbilics and the swallowtail. Cusps are contained in all of them. The parabolic umbilic is not self-dual. The regions where minima for potentials exist are very small. Getting 3-dimensional pictures of the 6-dimensional parabolic umbilic is described in [6].

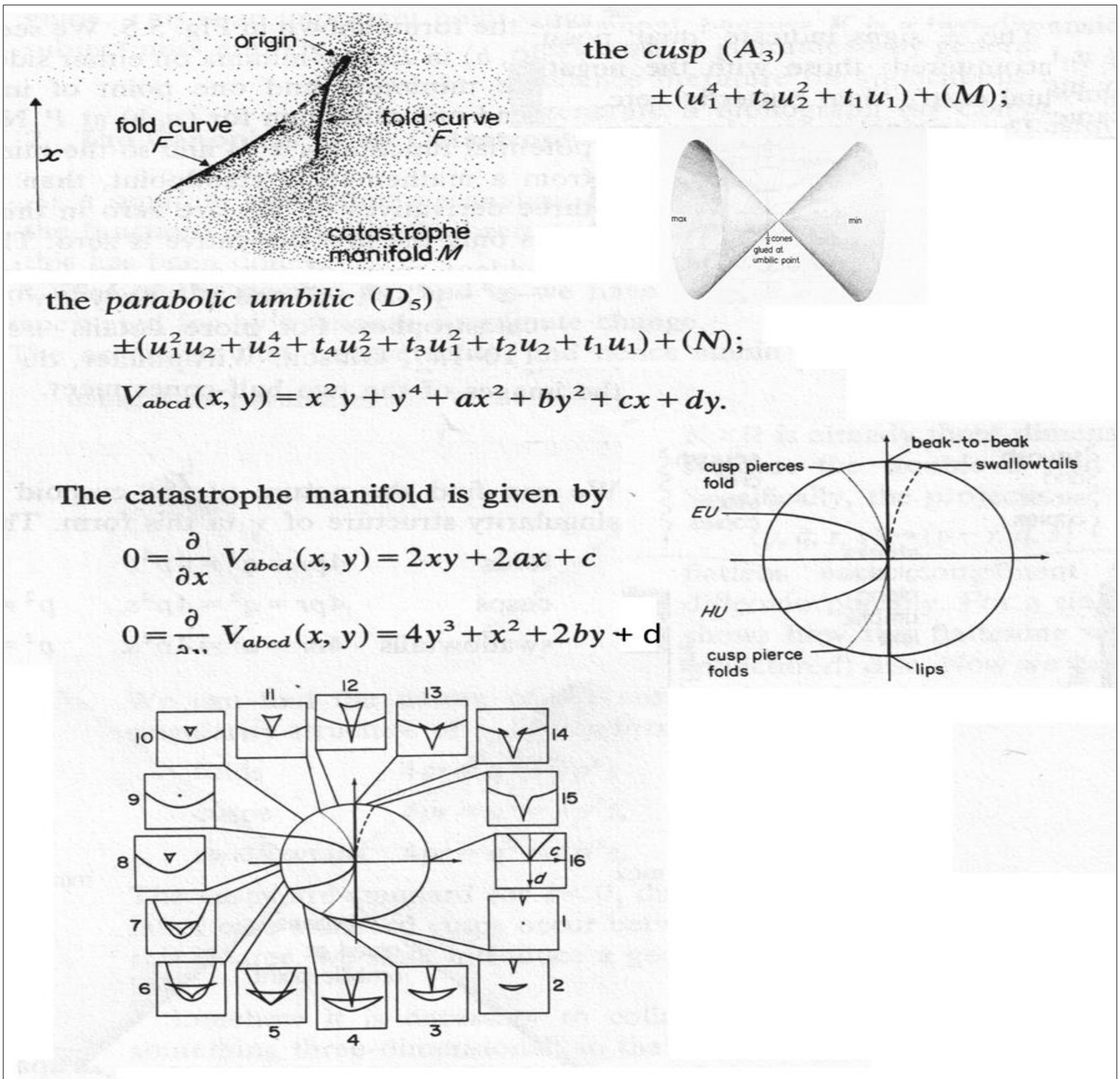

Figure 3 The cusp upper row, at right below the parabolic umbilic double cone, at left the equations of the the parabolic catstrophe, its potential function with variables and parameters, its manifold equations, at right below the cone, is shown how the bifurcations are generated, lower line some details for the evolving parabolic geometry in steps 1-16. Source: [5].

In catastrophe applications [5] buckling problems are treated, using the parabolic umbilic.

6. Cross ratios

The six sequence of color charges is presented by the complex six cross ratios (z,0;-1,∞) as invariants under Moebius transformations. In the usual, not the above D3 presentation, they are z for an identity map and the color charge red, octonian coordinate 1, EM(pot) energy; 1/z for inversions, a D3 reflection σ1, the color charge turquoise c(r), octonian coordinate 5, E(pot) energy; z/(z-1) for entropy, a D3 reflection ασ1 , the color charge green, octonian coordinate 2, E(heat) energy with phonons; (z-1)/z for rotatons, a D3 rotation α^2 of order 3, the color charge c(g), octonian coordinate 3, E(rot) energy with angular momentum and speed; (1-z) for time translations, a D3 reflection $\alpha^2\sigma1$, the color charge y = c(b), octonian coordinate 4, E(magn) magnetic energy; 1/(1-z) is for linear speeds and momentum, a D3 rotation α of order 3, the color charge blue b, octonian coordinate 6, E(kin) kinetic energy. The four tuples arise as factors from the symmetry S4 of the four permuted elements in the above cross ratio where D3 is obtained from S4 by factoring it with its normal Klein

CPT-subgroup Z2 xZ2 of order 4.

From last centuries publications of the author [4] is added a table:

Table 1 Lines are for six color charges symmetreis, coordinates, energies

In the first line of the table are the spherical SI coordinates, possibly 7- (not 8-)dimensional extended with exponential/polar coordinates. In the second line are the linear Pauli/Euclidean coordinates, in the third line a distribution of color charges to the SI coordinates. The fourth (fifth) line contains the D_3 (SU(2)/Pauli) MTs as cross ratios. Their matrix names are in the sixth line, together with the Einstein matrices. The following line is a numbering for a strong 6-fold integration series (not the Fano figures numbers which are for octonians). The next line contains the Planck numbers. Energy vectors are in the second to last line and the last line contains natural constants and three more operators, C (conjugation for quantum numbers), T (time reversal) and P (space parity) of physics.

r or $re^{i\varphi_1}$	φ	θ	ict	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$	$z \in \mathbb{R}$			
r	g	$\overline{g}$	$\overline{b}$	b	$\overline{r}$
z	$\frac{z}{-z-1}$	$\frac{-z-1}{z}$	$(-z-1)$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	-$\frac{1}{z}$	$-z$	z		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	$\alpha^2; \sigma_3$	$\alpha^2\sigma_1; id$	$\alpha; \delta$	$\sigma_1; id; \beta$
1	6	4	2	5	3
length λ_P	temp. T_P	dens. ρ_P	time t_P	ener. E_P	mass m_P
EM_{pot}	E_{heat}	E_{rot}	E_{magn}	E_{kin}	E_{pot}
c, e_0, ϵ_0	k,C	N_A,T	μ_0	h	γ_G, R_S,P

The pairing of coordinates in the HU as 15,23,46 is bifurcated after photons, EMI, spectral series can be emitted from atoms. For octonians the pairing is 15,46, 03 and 27.

The color charge coordinates are mapped as HU rays to xyz-space coordinates as shown in the hedgehog figure. The octonian and SU(3) 8-dimensional extension adds as complex cross product to the tables C^3 a fourth complex plane z4 = (e0 ,U(1)), a G-compass. Its (octonian) eigenvector e0 sets unit vectors for energy coordinates with measures attached. The octonian coordinate e7 is stereographic rolled up to a U(1) circle of EMI. The matrix G of order 6 generates many 6-cyclic sequencies, the general relativistic scaling factor for Schwarzschild metric is its radial scaled version.

7. Conclusion

Color charges as independent energies are introduced, not depending from quantum numbers for quarks as used in QCD. They serve as a new interaction acting for some quantas like gluons or quarks, and also for gravity in form of rgb-graviton whirls. There are many examples available which show that color charges come up in a complex 3-dimensional space C^3, extended by an energy plane from spactime. The GellMann 3x3-matrices of SU(3) are used as scaled projection maps, a catastrophe and models are presented for the complex 3-dimensional color charge space. Color charges can be called a force. Basic forces in physics are gravity to which they contribute the field quantums for its potential, the weak force, EM and electromagnetic force EMI which are not interacting with color charges, the strong force is interacting as mentioned above.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest.

References

[1] Kalmbach G. Orthomodular Lattices. – London New York: Academic Press. 1983; 390.

[2] Kalmbach HEG. MINT-Wigris. – MINT Verlag Bad Woerishofen. 2019.

[3] Kalmbach HEG. (2020): Gravity with color charges. - J. of Emerging Tresnds in Engineering and applied Sciences. 2020; 11(5): 183-189.

[4] Kalmbach HEG. MINT (Mathematik, Informatik, Naturwissenschaften, Technik), Volume 1-65. – Bad Wörishofen: MINT Verlag. 1997-2020.

[5] T Poston, I. Stewart. Catastrophe theory and its applications, Pitman, London. 1978.

[6] R Burton, A Rockwood. An inexpensive technique for displaying algebraically defined surfaces. - In: G.K. Francis, Graphic Techniques in Geometry and Topology. AMS Proc., Evanston, Illinois. April 1977.

[7] E. Schmutzer. Projektive einheitliche Feldtheorie, Harry Deutsch, Frankfurt. 2004.

[8] K Stierstadt. Physik der Materie, VCH, Weinheim. 1989.

[9] MINT-Wigris project (G. Kalmbach H.E.), in the internet under: researchgate.net.