Open Access Research Journal of
Science and Technology
Journals home page: https://oarjst.com/
ISSN: 2782-9960 (Online)

(RESEARCH ARTICLE)

On Jordan Generalized (σ,τ) - Higher Reverse Derivations on Rings

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Open Access Research Journal of Science and Technology, 2026, 16(02), 001–005

Publication history: Received on 21 January 2026; revised on 01 March 2026; accepted on 02 March 2026

Article DOI: https://doi.org/10.53022/oarjst.2026.16.2.0020

Abstract

The target of this schooling to display the notion of generalized (σ,τ)- higher reverse derivation of a ring Ł is rely content (σ,τ)- higher reverse derivation

Keywords: Generalized (σ,τ)- higher reverse derivation; Target; Higher reverse derivation; 2-torsion free ring

Mathematics Subject Classification(2000): 16W25, 16N80,16U80

1. Introduction

The idea of generalized (σ,τ)- higher reverse derivation, Jordan generalized (σ,τ)- higher reverse derivation & Jordan triple generalized (σ,τ)- higher reverse derivation (for short g(σ,τ) , J.g(σ,τ) & J.t.g(σ,τ)) resp. of a ring Ł substantial in non-commutative algebra. In[1] and[2] can the definition of 2-torsion free ring (for short 2- t) , approaching for more initiation [2&3] .

2. Jordan generalized (σ,τ)-Higher Reverse Derivations on Rings

Definition(2.1):

Permit Ҿ = $(Ҿ_i)_{i\in N}$ belongings on multilateral mappings of a ring Ł into same & σ,τ be two endomorphisms of Ł.

Then Ҿ is called a g(σ,τ) associated with the (σ,τ) – h.r.d.Ğ = $(ğ_i)_{i\in N}$ of Ł,∀ ʂ , ʐ ∈ Ł & n ∈ N

$$Ҿ_n(ʂʐ) = \sum_{z+v=n} Ҿ_z(\sigma^{n-z}(ʐ))\, ğ_v(\tau^{n-v}(ʂ)) \quad ...(1)$$

Ҿ is called a J.g(σ,τ).associated with the J.(σ,τ) – h.r.d.Ğ = $(ğ_i)_{i\in N}$, then

$$Ҿ_n(ʂ^2) = \sum_{z+v=n} Ҿ_z(\sigma^{n-z}(ʂ))\, ğ_v(\tau^{n-v}(ʂ)) \quad ...(2)$$

& Ҿ is called a J.t.g(σ,τ) associated with the J.t.(σ,τ) – h.r.d.Ğ = $(ğ_i)_{i\in N}$,then

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$$\text{Ꝕ}_n(\text{ȿɀȿ}) = \text{Ꝕ}_n(\text{ȿ})\text{ȿɀ} + \sum_{\substack{z<n \\ z+v+u=n}} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}))\, \check{g}_v(\sigma^u \tau^z(\text{ɀ}))\, \check{g}_u(\tau^{n-u}(\text{ȿ})) \ldots(3)$$

Lemma(2.2):

Permit $\text{Ꝕ} = (\text{Ꝕ}_i)_{i\in N}$ be a J.g(σ,τ) of Ł . Then $\forall$ ȿ , ɀ , ƀ $\in$ Ł & n $\in$ N

(a) $$\text{Ꝕ}_n(\text{ȿɀ} + \text{ɀȿ}) = \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ɀ}))\, \check{g}_v(\tau^{n-v}(\text{ȿ}) + \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}))\, \check{g}_v(\tau^{n-v}(\text{ɀ}))$$

(b) $$\text{Ꝕ}_n(\text{ȿɀƀ} + \text{ƀɀȿ}) = \text{Ꝕ}_n(\text{ƀ})\text{ȿɀ} + \sum_{\substack{z<n \\ z+v+u=n}} \text{Ꝕ}_z(\sigma^{n-z}(\text{ƀ}))\, \check{g}_v(\sigma^u \tau^z(\text{ɀ}))\, \check{g}_u(\tau^{n-u}(\text{ȿ})) +$$

$$\text{Ꝕ}_n(\text{ȿ})\text{ƀɀ} + \sum_{\substack{z<n \\ z+v+u=n}} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}))\, \check{g}_v(\sigma^u \tau^z(\text{ɀ}))\, \check{g}_u(\tau^{n-u}(\text{ƀ}))$$

(c) In individual , if Ł is a 2-t commutative

$$\text{Ꝕ}_n(\text{ȿɀƀ}) = \text{Ꝕ}_n(\text{ƀ})\text{ȿɀ} + \sum_{\substack{z<n \\ z+v+u=n}} \text{Ꝕ}_z(\sigma^{n-z}(\text{ƀ}))\, \check{g}_v(\sigma^u \tau^z(\text{ɀ}))\, \check{g}_u(\tau^{n-u}(\text{ȿ}))$$

Proof:

(a) subrogate ȿ + ɀ for ȿ in the Definition(2.1)(b),

$$\text{Ꝕ}_n((\text{ȿ}+\text{ɀ})(\text{ȿ}+\text{ɀ})) = \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}+\text{ɀ}))\, \check{g}_v(\tau^{n-v}(\text{ȿ}+\text{ɀ}))$$

$$= \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}))\, \check{g}_v(\tau^{n-v}(\text{ȿ})) + \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}))\, \check{g}_v(\tau^{n-v}(\text{ɀ})) + \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ɀ}))\check{g}_v(\tau^{n-v}(\text{ȿ})) + \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ɀ}))\check{g}_v(\tau^{n-v}(\text{ɀ})) \ldots(1)$$

On the other hand

$$\text{Ꝕ}_n((\text{ȿ}+\text{ɀ})(\text{ȿ}+\text{ɀ})) = \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}))\, \check{g}_v(\tau^{n-v}(\text{ȿ})) + \sum_{z+v=n} \text{Ꝕ}_z(\sigma^{n-z}(\text{ɀ}))\, \check{g}_v(\tau^{n-v}(\text{ɀ})) + \text{Ꝕ}_n(\text{ȿɀ} + \text{ɀȿ}) \ldots(2)$$

Comparing (1) and (2), outcome

(b) subrogate ȿ + ɀ for ȿ in Definition (2.1)

$$\text{Ꝕ}_n((\text{ȿ}+\text{ƀ})\text{ɀ}(\text{ȿ}+\text{ƀ})) = \text{Ꝕ}_n((\text{ȿ}+\text{ƀ}))(\text{ȿ}+\text{ƀ})\,\text{ɀ} + \sum_{\substack{z<n \\ z+v+u=n}} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}+\text{ƀ}))\, \check{g}_v(\sigma^u \tau^z(\text{ɀ}))\, \check{g}_u(\tau^{n-u}(\text{ȿ}+\text{ƀ}))$$

$$= \text{Ꝕ}_n(\text{ȿ})\,\text{ȿ}\,\text{ɀ} + \sum_{\substack{z<n \\ z+v+u=n}} \text{Ꝕ}_z(\sigma^{n-z}(\text{ȿ}))\, \check{g}_v(\sigma^u \tau^z(\text{ɀ}))\, \check{g}_u(\tau^{n-u}(\text{ȿ})) +$$

$$\text{Ꝕ}_n(\text{ƀ})\,\text{ȿ}\,\text{ɀ} + \sum_{\substack{z<n \\ z+v+u=n}} \text{Ꝕ}_z(\sigma^{n-z}(\text{ƀ}))\, \check{g}_v(\sigma^u \tau^z(\text{ɀ}))\, \check{g}_u(\tau^{n-u}(\text{ȿ})) +$$

$$\text{Ҫ}_n(\text{ş})\,\text{ŧ}\,\zeta + \sum_{z+v+u=n}^{z<n} \text{Ҫ}_z(\sigma^{n-z}(\text{ş}))\,\check{g}_v(\sigma^u \tau^z(\zeta))\,\check{g}_u(\tau^{n-u}(\text{ŧ})) +$$

$$\text{Ҫ}_n(\text{ŧ})\,\text{ŧ}\,\zeta + \sum_{z+v+u=n}^{z<n} \text{Ҫ}_z(\sigma^{n-z}(\text{ŧ}))\,\check{g}_v(\sigma^u \tau^z(\zeta))\,\check{g}_u(\tau^{n-u}(\text{ŧ})) \quad \ldots(1)$$

On the other hand

$$\text{Ҫ}_n((\text{ş}+\text{ŧ})\zeta(\text{ş}+\text{ŧ})) = \text{Ҫ}_n(\text{ş})\,\text{ş}\,\zeta + \sum_{z+v+u=n}^{z<n} \text{Ҫ}_z(\sigma^{n-z}(\text{ş}))\,\check{g}_v(\sigma^u \tau^z(\zeta))\,\check{g}_u(\tau^{n-u}(\text{ş})) +$$

$$\text{Ҫ}_n(\text{ŧ})\,\text{ŧ}\,\zeta + \sum_{z+v+u=n}^{z<n} \text{Ҫ}_z(\sigma^{n-z}(\text{ŧ}))\,\check{g}_v(\sigma^u \tau^z(\zeta))\,\check{g}_u(\tau^{n-u}(\text{ŧ})) + \text{Ҫ}_n(\text{ş}\,\zeta\,\text{ŧ} + \text{ŧ}\,\zeta\,\text{ş}) \quad \ldots(2)$$

Comparing (1) and (2), outcome

(c) By (b) & since Ł is a 2-t commutative , outcome

Definition (2.3):

Permit $\text{Ҫ} = (\text{Ҫ}_i)_{i\in N}$ be a J.g(σ,τ) of Ł . Then $\forall\ \text{ş}, \zeta \in$ Ł & $n \in N$

$$\Psi = \text{Ҫ}_n(\text{ş}\zeta) - \sum_{z+v=n} \text{Ҫ}_z(\sigma^{n-z}(\zeta))\,\check{g}_v(\tau^{n-v}(\text{ş}))$$

Lemma (2.4):

Permit $\text{Ҫ} = (\text{Ҫ}_i)_{i\in N}$ be a J.g(σ,τ) of Ł. Then $\forall\ \text{ş}, \zeta, \text{ŧ} \in$ Ł & $n \in N$:

(i) $\Psi_n(\text{ş}, \zeta) = -\Psi_n(\zeta, \text{ş})$

(ii) $\Psi_n(\text{ş} + \zeta, \text{ŧ}) = \Psi_n(\text{ş}, \text{ŧ}) + \Psi_n(\zeta, \text{ŧ})$

(iii) $\Psi_n(\text{ş}, \zeta + \text{ŧ}) = \Psi_n(\text{ş}, \zeta) + \Psi_n(\text{ş}, \text{ŧ})$

Remark (2.5) :

Permit $\text{Ҫ} = (\text{Ҫ}_i)_{i\in N}$ is a g(σ,τ) of Ł if & only if $\Psi_n = 0$, $\forall\ n \in N$.

Theorem (2.6):

Permit $\text{Ҫ} = (\text{Ҫ}_i)_{i\in N}$ be a J.g(σ,τ) of Ł. Then $\Psi_n = 0$, $\forall\ n \in N$.

Proof :

By Lemma (2.2) (i)

$$\text{Ҫ}_n(\text{ş}\zeta + \zeta\text{ş}) = \sum_{z+v=n} \text{Ҫ}_z(\sigma^{n-z}(\zeta))\,\check{g}_v(\tau^{n-v}(\text{ş})) + \sum_{z+v=n} \text{Ҫ}_z(\sigma^{n-z}(\text{ş}))\,\check{g}_v(\tau^{n-v}(\zeta)) \ldots(1)$$

On the other hand :

$$\text{Ҫ}_n(\text{ş}\zeta + \zeta\text{ş}) = \text{Ҫ}_n(\text{ş}\zeta) + \sum_{z+v=n} \text{Ҫ}_z(\sigma^{n-z}(\text{ş}))\,\check{g}_v(\tau^{n-v}(\zeta)) \ldots(2)$$

Comparing (1) & (2)

$$\text{Ҫ}_n(\text{ş}\zeta) = \sum_{z+v=n} \text{Ҫ}_z(\sigma^{n-z}(\zeta))\,\check{g}_v(\tau^{n-v}(\text{ş}))$$

$$Ҿ_n(ȿɀ) - \sum_{z+v=n} Ҿ_z(\sigma^{n-z}(ɀ))\, ğ_v(\tau^{n-v}(ȿ)) = 0$$

By Definition (2.3) , we get it .

Corollary (2.7) :

Each J. g(σ,τ).of Ł is a g.(σ,τ) .

Proof :
By Theorem (2.6), $\Psi_n = 0$, $\forall$ n$\in$ N & by Remark (2.5), outcome

Proposition (2.8) :

Each J.g(σ,τ).of a 2-t Ł is a J.t.g(σ,τ) .

Proof :

Replace ȿɀ + ɀȿ for ɀ in Lemma (2.2) (i)

$$Ҿ_n= (ȿ\,(ȿɀ + ɀȿ) + (ȿɀ + ɀȿ)\,ȿ)$$

$$= Ҿ_n(ɀ)\,ȿ^2 + \sum_{\substack{z<n\\ z+v+u=n}} Ҿ_z(\sigma^{n-z}(ɀ))\, ğ_v(\sigma^u \tau^z(ȿ))\, ğ_u(\tau^{n-u}(ȿ)) +$$

$$2(Ҿ_n(ȿ)\,ȿ\,ɀ + \sum_{\substack{z<n\\ z+v+u=n}} Ҿ_z(\sigma^{n-z}(ȿ))\, ğ_v(\sigma^u \tau^z(ɀ))\, ğ_u(\tau^{n-u}(ȿ))) +$$

$$Ҿ_n(ȿ)\,ɀ\,ȿ + \sum_{\substack{z<n\\ z+v+u=n}} Ҿ_z(\sigma^{n-z}(ȿ))\, ğ_v(\sigma^u \tau^z(ȿ))\, ğ_u(\tau^{n-u}(ɀ)) \ldots (1)$$

On the other hand:

$$Ҿ_n= (ȿ\,(ȿɀ + ɀȿ) + (ȿɀ + ɀȿ)\,ȿ)$$

$$Ҿ_n(ɀ)\,ȿ^2 + \sum_{\substack{z<n\\ z+v+u=n}} Ҿ_z(\sigma^{n-z}(ɀ))\, ğ_v(\sigma^u \tau^z(ȿ))\, ğ_u(\tau^{n-u}(ȿ)) +$$

$$Ҿ_n(ȿ)\,ɀ\,ȿ + \sum_{\substack{z<n\\ z+v+u=n}} Ҿ_z(\sigma^{n-z}(ȿ))\, ğ_v(\sigma^u \tau^z(ȿ))\, ğ_u(\tau^{n-u}(ɀ)) + 2\,Ҿ_n(ȿ\,ɀ\,ȿ) \ldots (2)$$

Comparing (1) , (2) & since Ł is a 2-t , outcome

3. Conclusion

In this paper we defined the concept of generalized (σ,τ)- higher reverse derivation , Jordan generalized (σ,τ)- higher reverse derivation and Jordan triple generalized (σ,τ)- higher reverse derivation on Rings we can find the relation between the concept of generalized (σ,τ)- higher reverse derivation and Jordan generalized (σ,τ)- higher reverse derivation with certain conditions . According to the previous results, the future study in this paper will define on Γ– rings .

Compliance with ethical standards

Disclosure of conflict of interest

There was no conflict of interest to be disclosed.

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