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Bayesian approximation for parameterized KALMAN filter for investigation and simulation of unknown noise variance trajectory following in state space models with different noise distributions

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Abstract

Bayesian approach can be used for parameter identification and extraction in state space models and its ability for analyzing sequence of data in dynamical system is proved in different literatures. In this paper, Bayesian approach for approximation of variances in measurement noise with KALMAN filter is applied for estimation of the dynamical state and measurement data in discrete dynamical system. Detection of uncertainty and estimation of those can be done simultaneously with adaptive KALMAN filter. This algorithm at each step time estimates noise variance and state of system with KALMAN filter. Then, approximation is formed at each step separately and at each step sufficient statistics of the state and noise variances are computed with a fixed-point iteration of a KALMAN filter. For showing influence of variance in measurement data on algorithm different simulations is applied. First, effect of variance and its distribution on detection performance is simulated in KALMAN filter without Bayesian formulation. Then simulation is applied to KALMAN filter with ability of variance tracking of measurement data.in these simulations, influence of distribution of measurement data in each step is estimated and true variance of data is obtained by algorithm and is compared in different scenarios. Afterwards, one typical modeling of nonlinear state space model with inducing noise measurement is simulated by this approach. Finally, the performance and the important limitations of this algorithm in these simulations are explained.

Keywords: Adaptive filtering methods; KALMAN filtering; Variance tracking; Fluctuation

1. Introduction

The KALMAN Filter (KF) can be estimated dynamical state from noisy measurements. In this method, dynamic and measurement processes can be approximated by linear Gaussian state space models [1]. This model is a practical model in engineering due to modelling of various noises. The extended KALMAN filter (EKF) and the unscented KALMAN filter (UKF) encompasses this method to nonlinear dynamical states and measurement by forming a Gaussian approximation to the posterior state distribution in modelling [2]–[5]. A serious constraint in these filters is that they assume priori knowledge of the measurement and the parameters of dynamical model, including the noise statistics status. The exact knowledge of the parameters and especially the noise statistics characteristics is not obvious and we don't have exact information about it in many practical situations. Examples of such applications are integrated GPS positioning systems or fault detection systems [6], [7]. For solving this problem and solving uncertain parameters in model there is a different algorithm that among those, adaptive filters are common. This approach can be estimated noise statistics characteristic and also estimation of dynamical states and measurement can be done simultaneously [8]-[9]. In literatures, different adaptive filtering approach is divided to Bayesian and correlation analysis and also covariance matching methods [8].

In different signal processing applications, there are many sources of interference and noise in systems and in these conditions, efficiency of algorithm for computation and estimation is vital. Bayesian approach is more common from

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other approaches and in different computational signal processing, this approach is used. As said before, estimation of uncertainty with dynamical states is important in filtering problems and Bayesian approach is a strong method for approximation of posterior status of these disturbances. Some references like [10] and [11] is used multiple model methods and state augmentation approach which is developed from Bayesian formulations. On the other hand, references like [12,13 and 14] is used and developed Bayesian approach based on approximation of posterior distribution and one of the important advantages of this algorithm is related to low computational cost time. Moreover ref [14] is developed an approach for recursive Bayesian inference and its approach is suitable for signal processing applications.

In recent years, approximation algorithm for linear and nonlinear state space models with unknown and varying variances is proposed. In ref [15], a KALMAN smoother with Variational structure is proposed for approximation of stationary noises. On the other hand, in ref [16], a fixed form approach for models with time varying variances is proposed. One disadvantage of its approach is preparation of exact model of dynamical system and also statistical information should be accurate and available for algorithm.

In ref [17] Bayesian adaptive KALMAN filter is formulated and it can be used for variances in measurement and with dynamical state. But in this paper, we developed a series of simulation in conditions when variance of measurement has different distribution and statistical characteristics and then performance of approach for following and tracking this variance is modelled. Also, a nonlinear state space model is applied to these approach and estimation of states and measurement of system with adaptive KALMAN filter is investigated. Finally, the paper is structured as follows.

In this paper, first overall structure of algorithm for KALMAN filter is shown schematically. In many references, this method is used extensively regarding the approximation the joint posterior distribution of the state and the noise variances. Section 3, is related to problem formulation of adaptive KALMAN filter and steps for update and estimation in this approach is explained. Next, in section of experimental results, a series of simulation is applied and accordingly performance of approach for estimation of variance is simulated. Finally, in a nonlinear state space model for pendulum this approach is applied and estimation of measurement for this model is used.

2. Overall structure of algorithm

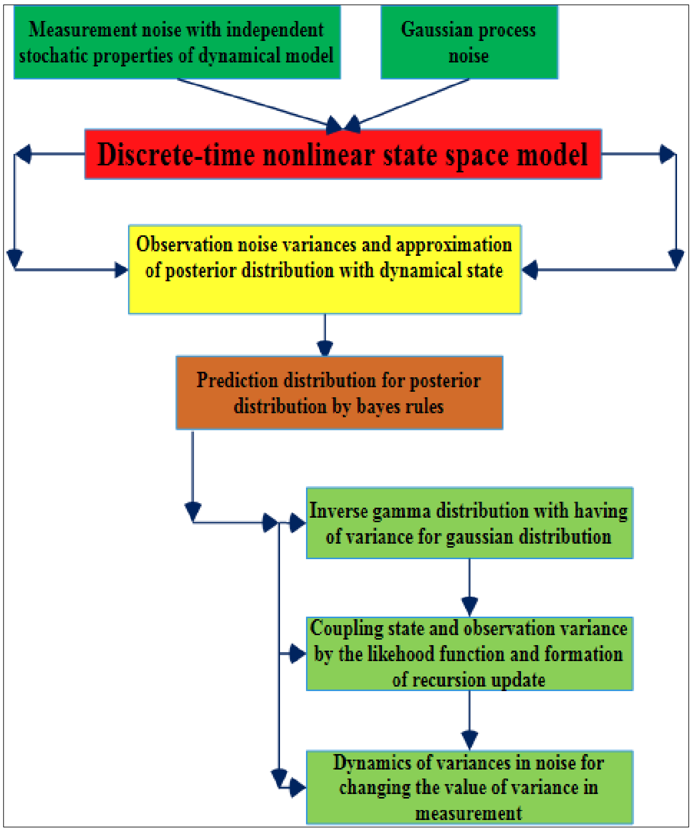

Figure 1 Overall structure of algorithm

In this section overall steps of algorithm from dynamical state formation and modelling to using of Bayesian approach for obtaining the recursion update for posterior estimation is shown in Fig. 1.

3. Problem Formulation

Summary formulation of algorithm is explained in blow second sections.

3.1. Overall modelling of dynamical system

The discrete-time linear state space model can be considered here by (1)

$$x_k = A_k x_{k-1} + q_k$$
$$y_k = H_k x_k + r_k \quad \ldots\ldots\ldots\ldots (1)$$

Where $q_k \sim N(0, Q_k)$ is the Gaussian process noise, $r_k \sim N(0, \Sigma_k)$ is the measurement noise with assumed covariance Σ_k, and the initial state has a prior Gaussian distribution $x0 \sim N(m(0), P(0))$.

The measurement y_k is a d-dimensional vector and the state x_k is an n-dimensional vector. Time is shown by k in the matrices A_k, H_k, Q_k, as well as the parameters of the initial state m_0, P_0 are assumed to be known in initial condition.

Now, departing from the case of standard KALMAN filter, observation noise variance parametersσ_k^2, i = 1, . . ., d, are stochastic with independent dynamic models. We denote the diagonal covariance matrix by Σ_k= diag (σ_2... σ_2).

The construction of a suitable dynamical model of the observation noise variances will be discussed, and at this stage it is denoted generically by p (Σ_k | Σ_{k-1}). Dynamic models of the states and the variance parameters are assumed independent according to the (2).

$$p\left(x_k, \sum k | x_{k-1}, \sum k-1\right) = p\left(x_k \mid x_{k-1}\right) p\left(\sum k \mid \sum k-1\right) \ldots\ldots (2)$$

The goal of Bayesian optimal filtering of the above model is to compute the posterior distribution p (x_k, Σ_k | y1: k). Generally, the well-known recursive solution to this filtering problem consists of the following steps [17].

Initialization: The recursion starts from the prior distribution p (x_0, Σ_0).

Prediction: The predictive distribution of the state x_k and measurement noise covariance Σ_k is given by the Chapman-Kolmogorov equation. Chapman – Kolmogorov equation compute marginal distribution of nuisance variable statistics characteristics. This identity integrates the joint probability distributions on different probability space on a stochastic process.

Update: Given the next measurement y_k, the predictive distribution above is updated to a posterior distribution by the Bayes' rule that is written in (3).

$$p\left(x_k, \sum y_{1:k}\right) \propto p\left(y_k \mid x_k, \sum k\right) p\left(x_k, \sum k \mid y_{1:k-1}\right) \ldots\ldots\ldots (3)$$

In [17] the recursion and suitable dynamics for the observation noise variances for the posterior update is proposed.

3.2. Bayesian inference formulation

Conditional distribution for x_{k-1} and Σk−1 is computed from the measurements y1... yk−1. An Independent Inverse-Gamma distribution is modeled as follows in equation (4). In this stage a conjugate prior assumes a closed form expression for the posterior update. Furthermore, state and observation noise variance coupled through the likelihood$p(y_k|x_k, \sum_k)$.

$$\mathrm{P}\ (x_{k-1}, \sum k-1 | y_{1:k-1}) = N(x_{k-1}\,|m_{k-1}, p_{k-1}) \times \prod_{i=1}^{d} inv-gamma(\sigma_{k-1}^2 | \alpha_{k-1,i}^{-}, \beta_{k-1,i}), \ldots \ldots (4)$$

This approximation is chosen, because the Inverse- Gamma distribution is the prior distribution. Also, using an Inverse-Gamma model for variances of Gaussian models is common in Bayesian analysis [17] because the dynamics of the state and observation noise variances are assumed to be independent.

It should be said in the posterior update step, the state and observation noise variance parameters will be coupled through the like hood distribution p ($y_k \| x_k, \Sigma_k$).

By forming the standard variation Bayesian (VB) approach [16]-[17] the finalized prediction updated cycle is obtained as follows in the (5)-(10).

$$Q_x(x_k) = N(x_k|m_k, P_k) \ldots\ldots\ldots\ldots\ldots (5)$$

$$Q_x(\sum k) = \prod_{i=1}^{d} inv - gamma(\sigma_{k-1}^2 \,|\alpha_{k-1,i}^-, \beta_{k-1,i}) \ldots\ldots\ldots (6)$$

$$p_k = p_k^- - p_k^- H_K^T (H_K p_k^- H_K^T + \textstyle\sum_K^\Lambda)^{-1} H_K p_k^- \ldots\ldots\ldots (7)$$

$$m_k = m_k^- + p_k^- H_k^- \left(H_k p_k^- H_k^T + \Sigma\mathrm{k}\right)^{-1} \times \left(y_k H_k m_k^-\right) (8) \quad \alpha_{k,i}^- = \tfrac{1}{2} + \alpha_{k-1,i}^- \ldots\ldots\ldots\ldots\ldots (9)$$

$$\beta_{k,i} = \beta_{k-1,i} + 1/2\left[\left(\mathrm{y_k} H_k m_k\right)^2 + \left(H_k p_k^- H_k^T\right)\right] \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Also, dynamic model of noise variance usually is not defined but it can be modeled by approximate posteriors. First in algorithm expected measurement noise precisions is considered constant, and then their variances are increased by a factor of ρ (ρ ∈ (0, 1]). This is obtained by following as in (11) and (12).

$$\alpha_{k,i}^- = \rho_i \alpha_{k-1,i}^- \ldots\ldots\ldots\ldots\ldots (11)$$

$$\beta_{k,i} = \rho_i \beta_{k-1,i} \ldots\ldots\ldots\ldots (12)$$

In these equations ρ=1 correspond to stationary variances and lower values increase their assumed fluctuation. In the modeling if the cross correlation between the prediction and observation error is ignored, then covariance becomes diagonal matrix and in many practical situations it is a proper assumption.

In the KALMAN filter joint posterior of state and observation noise variance can be found in a fixed iteration update. Then new expected noise covariance is computed.

4. Experimental Results

In this section, different simulation results are explained. First conditions when noise variance cannot be estimated by approach are presented in two different situations. Then, simulation results is done for KALMAN filter with ability of approximation for variance in measurement of data and influence of variance in measurement and its distribution is discussed. Also, in simulation results the variance of measurement is increased to show the effect of variance in this modeling approach. This artificial data has time varying error and also has unknown time varying variance. An example of the time varying nature of the errors involved is the initialization of the sensor error states.

4.1. Adaptive KALMAN filter without ability of approximation of variances

The first simulated data is shown in Fig. 2 and it is related to measurement of system with its estimation and then estimation of dynamical states through this KALMAN filter is shown in Fig. 3. Finally, estimated variance trajectory with true variance is compared and plotted in Fig .4. Because of wrong initial condition in variance the well matching is not obtained. However, the amplitude of this distribution in middle of time step is increased but it has more fluctuation. Also, the default variation of true variance is as follows. First in the simulation the measurement noise has the variance of o.2 and in the time step of 100 the variance quickly is increased to 1.45 and around time 200; it again quickly decrease to value 0.7. But, in the Variation Bayesian adaptive KALMAN filter the transition probabilities can be chosen in such a way that probability of switching mode from one mode to another is matched to the variation of variance with some try and error.

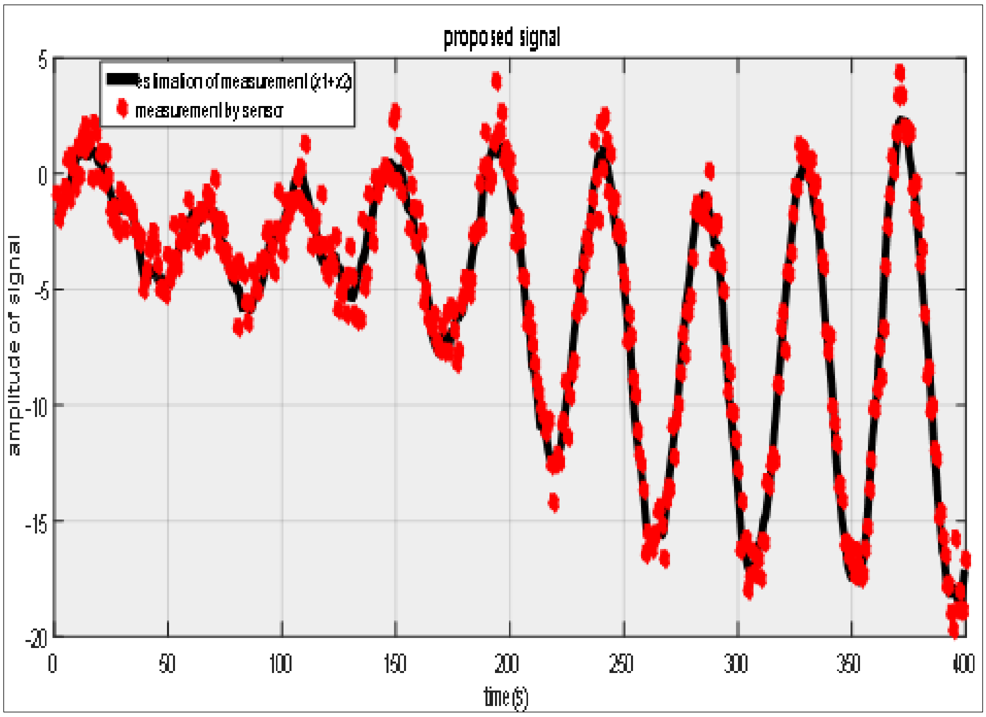

Figure 2 Estimated measurement data with KALMAN filter

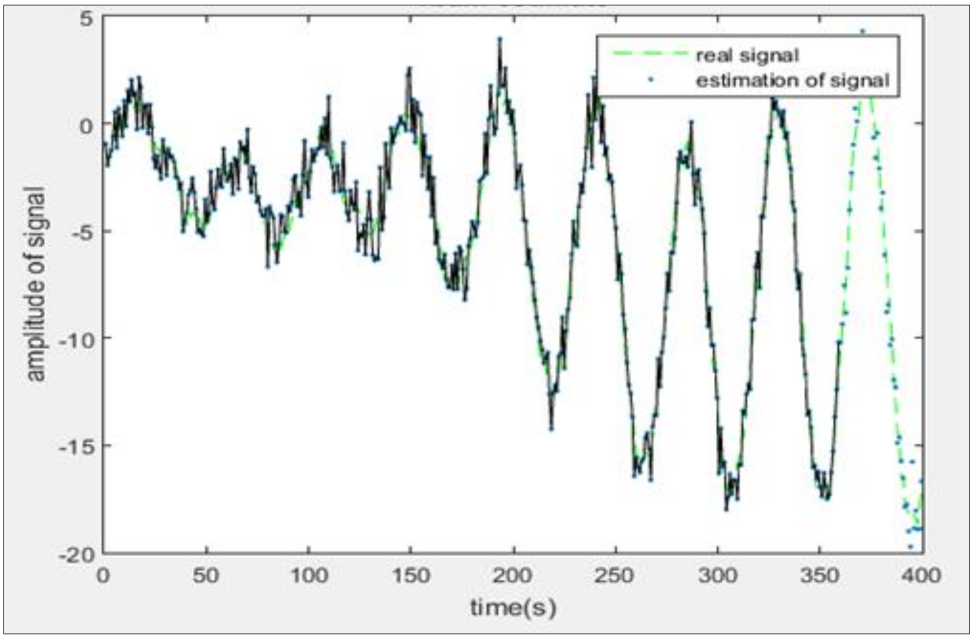

Figure 3 Estimation of dynamical states through KALMAN filter

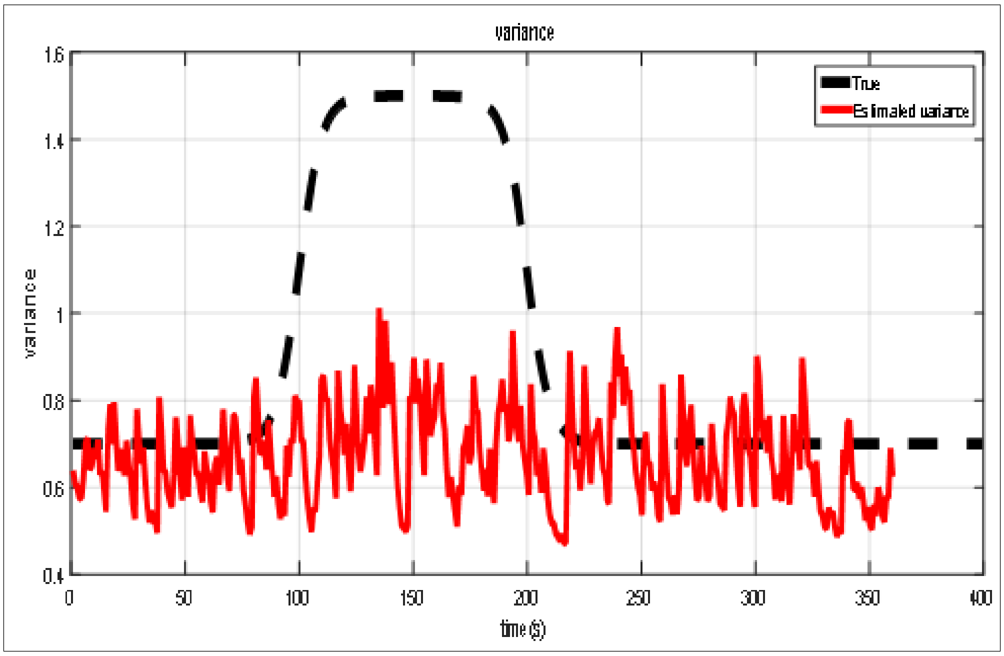

Figure 4 Comparison of true and estimated variance trajectory

In Fig .5 measurement data of another dynamical system is plotted with its estimation and also this simulation has more noise from former simulation. Next, dynamical state estimation with this approach is plotted in Fig.6 and finally the performance of KALMAN filter without ability of estimation of variance is plotted in Fig.7. Comparison of former simulation is in second simulation, due to the higher variance in measurement data the performance of KALMAN filter for variance following is poor and this makes that variation Bayesian for approximating of noise and variance is more important.

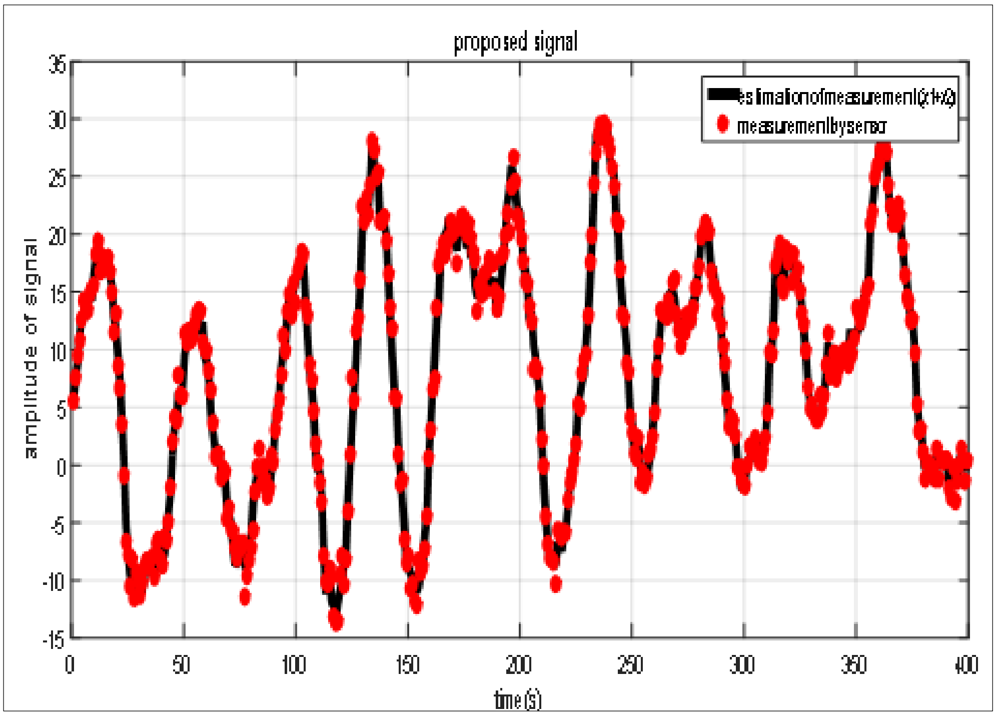

Figure 5 Estimated measurement data with KALMAN filter

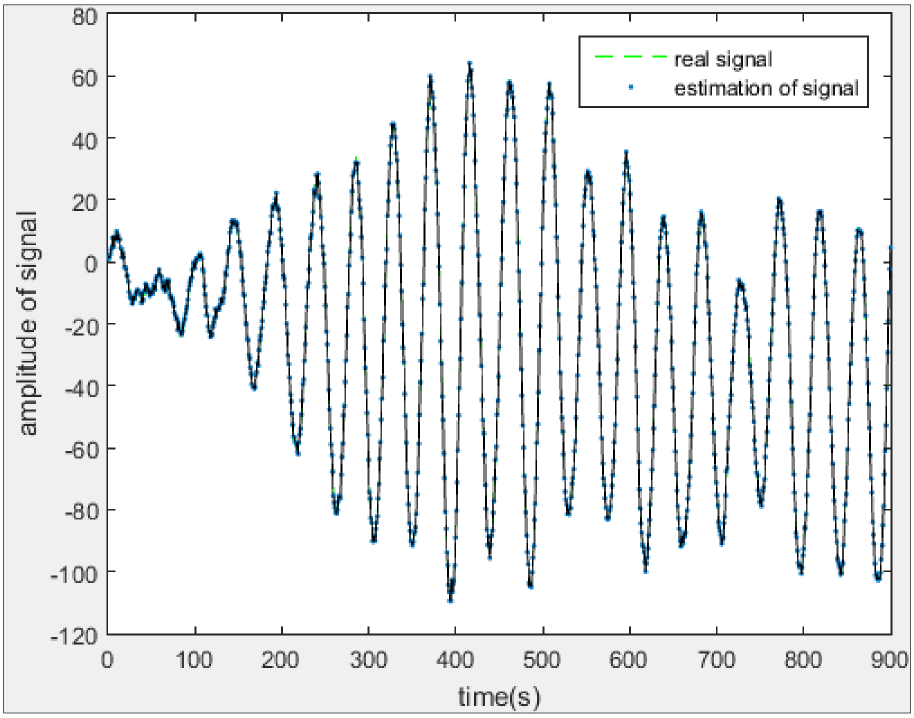

Figure 6 Estimation of dynamical states through KALMAN filter

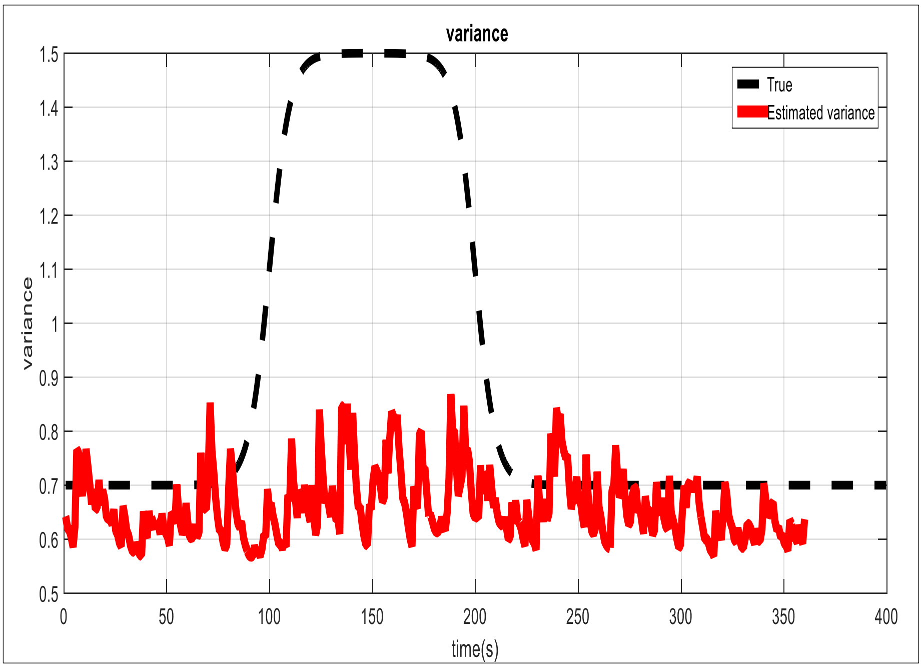

Figure 7 Comparison of true and estimated variance trajectory

4.2. Adaptive KALMAN filter with ability of approximation of variances with erupted initial condition

In this section KALMAN filter has ability of variance tracking of measurement data and it uses of Variational Bayesian approach. First Variance trajectory in different initial conditions is plotted and in this situation again the variance following is erupted with irreverent initial condition. This irregularity in initial condition of approach has important effect on variance tracking because the resulted error due to this condition cannot be removed in short step of algorithm and it has given deviation from exact variance trajectory and these conditions in two different simulations is applied.

Estimation of measurement and dynamical states with this approach for two different simulations is plotted in Fig.8 and Fig.9 and respected trajectory following is plotted in Fig.10 and Fig.11 respectively.

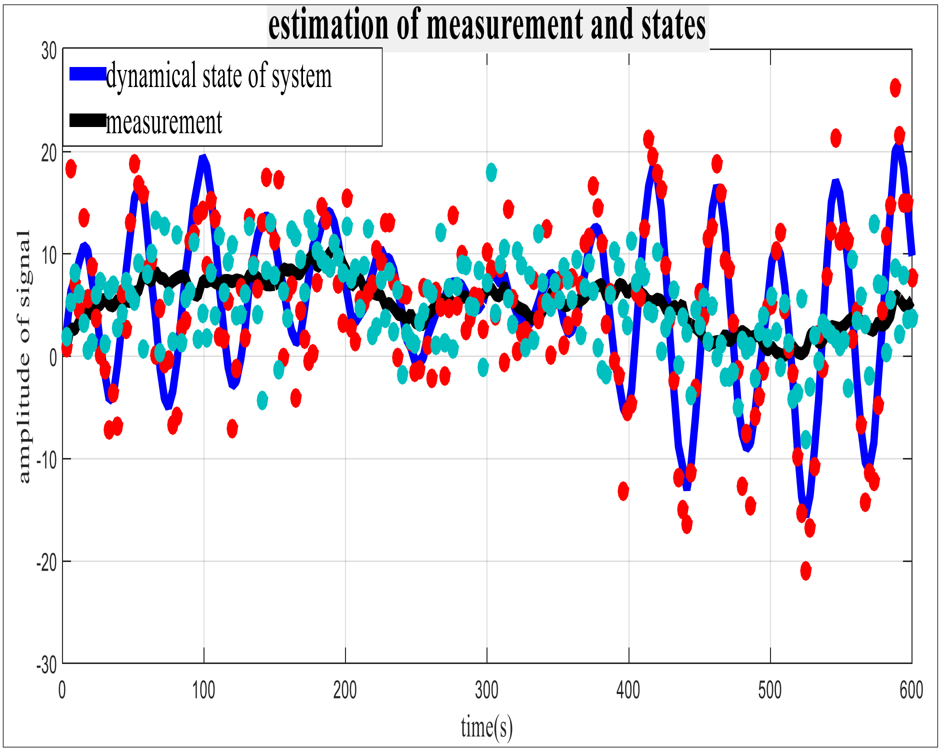

Figure 8 Estimated measurement data and dynamical states with KALMAN filter with Bayesian approach

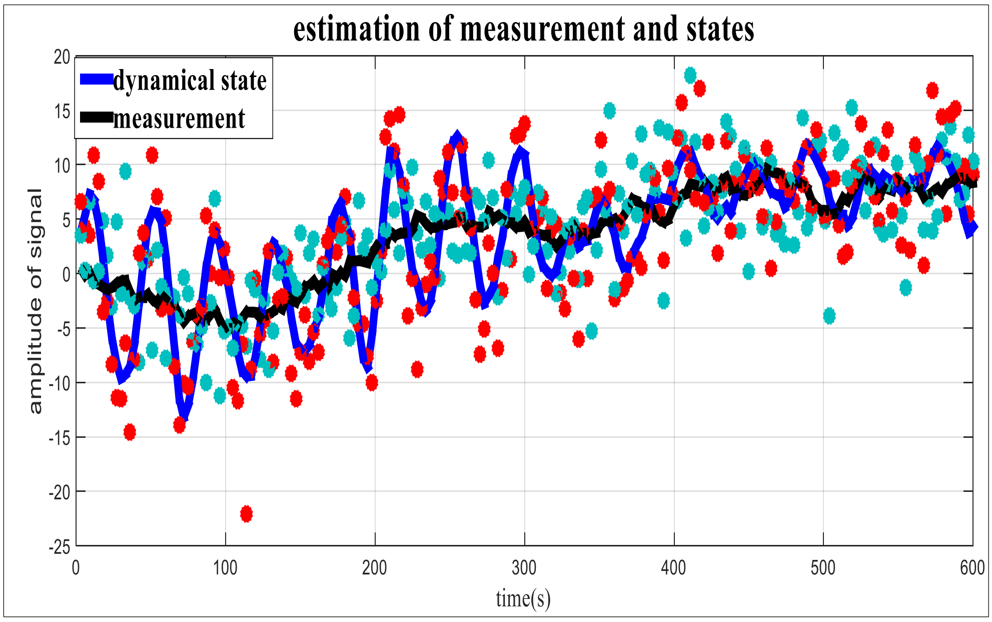

Figure 9 Estimated measurement data and dynamical states with KALMAN filter with Bayesian approach and more variance

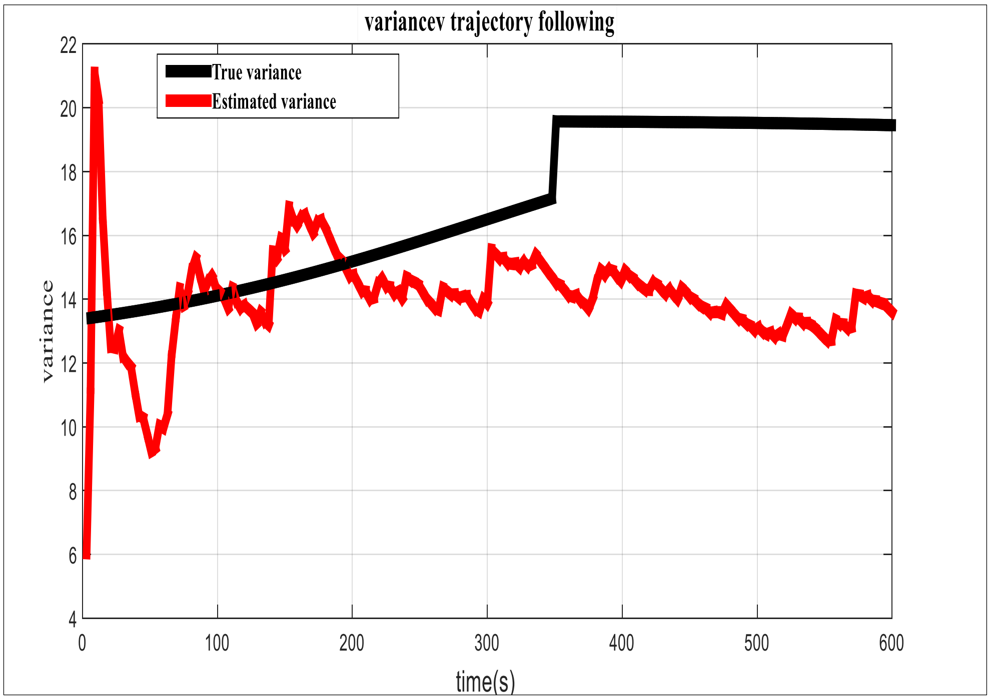

Figure10 Comparison of true and estimated variance trajectory with KALMAN filter with Bayesian approach

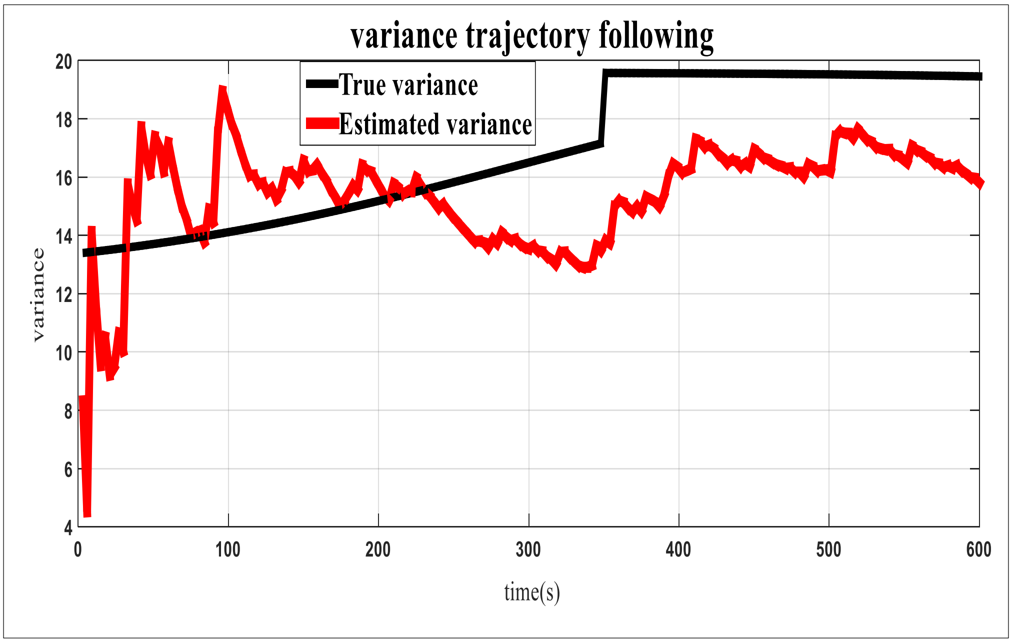

Figure 11 Comparison of true and estimated variance trajectory with KALMAN filter with Bayesian approach

4.3. Adaptive KALMAN filter with ability of approximation of variances

In this section, different simulations for developing the ability of KALMAN filter for variance trajectory following is investigated. Accordingly, in simulation the variance of measurement is increased and three simulations are examined in this section. The estimated measurement of data and dynamical states is plotted in Fig. 12 to Fig. 14 for each simulation and comparison of true and estimated variance trajectory following are plotted in Fig. 15 to Fig. 17 respectively.in respected to simulation results is evident this approach has poor performance for trajectory following of data when the variance is high.

Although this approach use approximation to the variance distribution and forms Gaussian state distribution conditionality in each time step but in high dimensional data with irreverent variance structure the assumption of center of limit for modeling of this approach is not practical well. Furthermore, due to the recursive nature of the filter estimation, the performance of the filter is dependent on a priori estimate. This means that the adaptive filter is not entirely self-tuning so we should consider the dimension of data for using this approach.

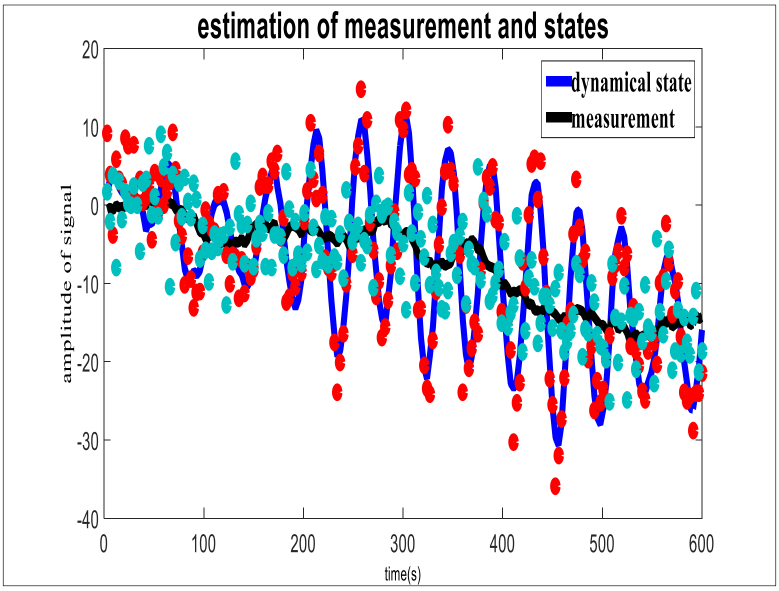

Figure 12 Estimated measurement data and dynamical states with KALMAN filter with Bayesian approach in case one

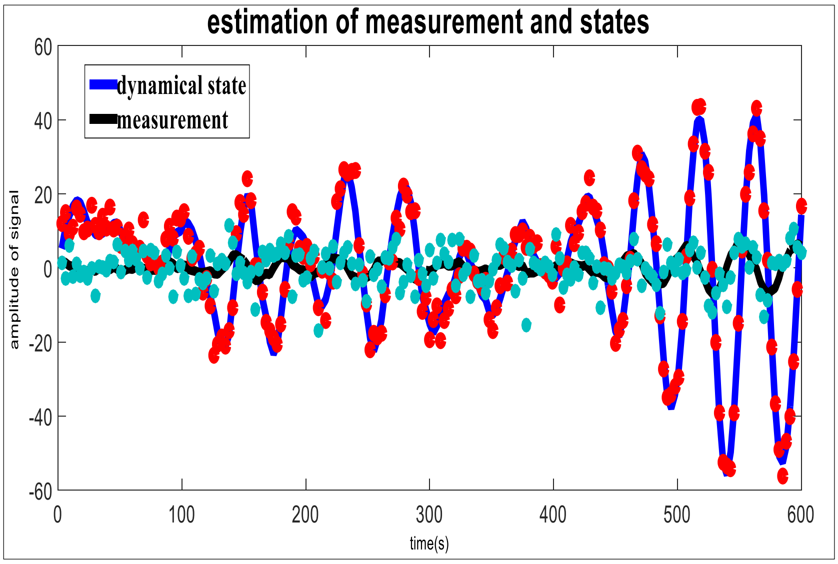

Figure 13 Estimated measurement data and dynamical states with KALMAN filter with Bayesian approach in second case

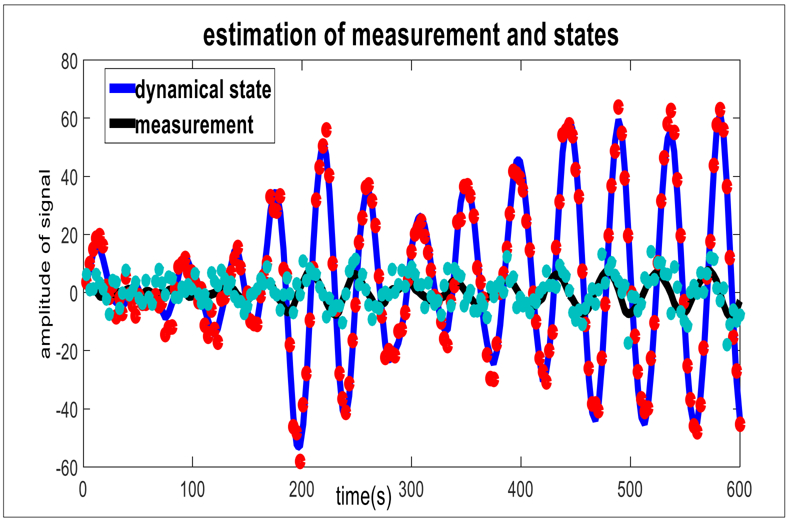

Figure 14 Estimated measurement data and dynamical states with KALMAN filter with Bayesian approach in third case

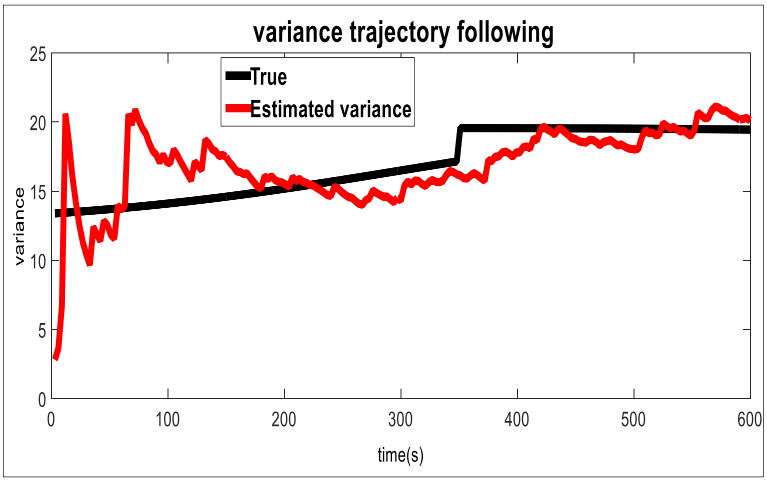

Figure 15 Comparison of true and estimated variance trajectory with KALMAN filter with Bayesian approach

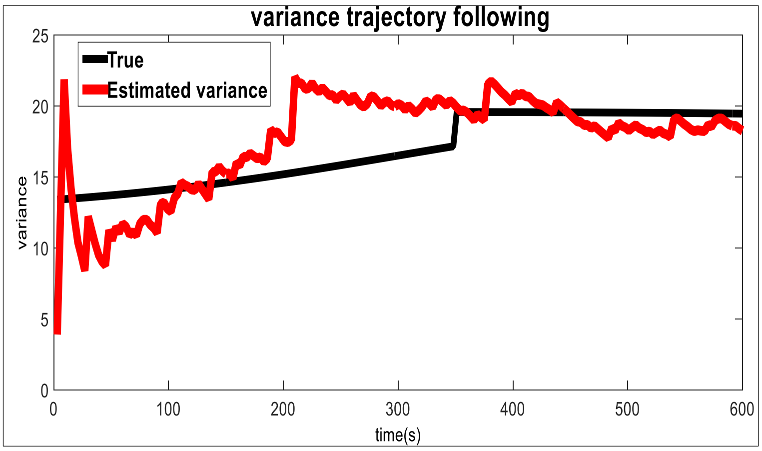

Figure 16 Comparison of true and estimated variance trajectory with KALMAN filter with Bayesian approach

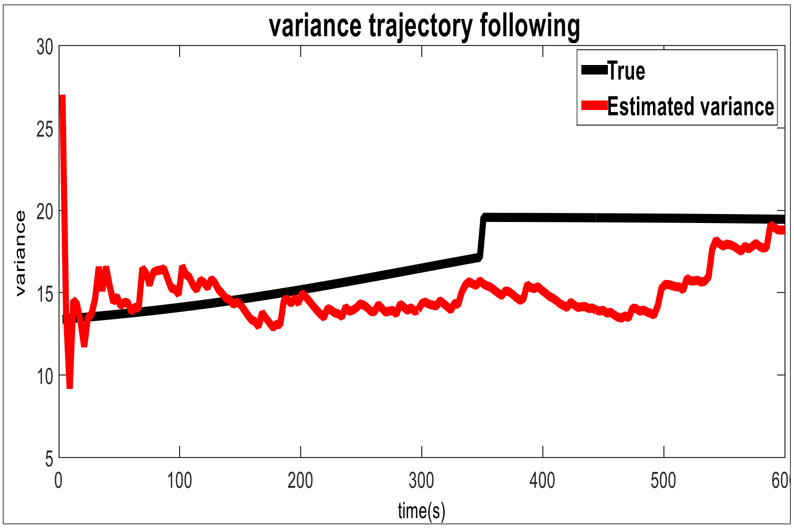

Figure 17 Comparison of true and estimated variance trajectory with KALMAN filter with Bayesian approach

5. Second modelling for Simple Pendulum with Noise disturbance

In this section the continuous-discrete sequential is applied to estimation of a partially measured simple pendulum model which is distorted by a random noise term. The stochastic differential equation for the angular position of a simple pendulum, which is distorted by random white noise accelerations w(t) with spectral density q can be written as in (13).

$$\frac{d^2x}{dt^2} + a^2 \sin(x) = w(t) \quad \text{........(13)}$$

If we define the state as x = (x dx/dt) and it is changed to state space form the model can be written as (14) and (15) [7].

$$\frac{dx_1}{dt} = x\,2 \quad \text{..........} \quad (14)$$

$$dx\,2 = -a^2 \sin(x\,1)dt + d\beta \quad \text{............(15)}$$

Assume that the state of the pendulum is measured once per unit time and the measurements are disturbed by Gaussian measurement noise with an unknown variance σ2 then a suitable model in this case can be written as in equation (16).

$$y_k = N\big(x_1(t_k)\big), \sigma^2)$$

$$\sigma^2 = INV - X^2(\nu_o, \sigma_o^2) \quad \dots\dots\dots..(16)$$

In Fig. 18 this model with measurement data, actual and estimated signal are plotted. According to the Fig. 18 it is evident that the estimation of signal is most generated in area with high concentration of data and in this area the correlation of our data is higher so this algorithm can detect this important information for producing and estimation of our signal. In this situation when high level of noise is inputted in measurement data this algorithm cannot follow the true signal well and this simulation is plotted in Fig. 19. In summary, when this method is chosen dimensional and variance of experiment should be considered and limitation of this approach to this condition is investigated.

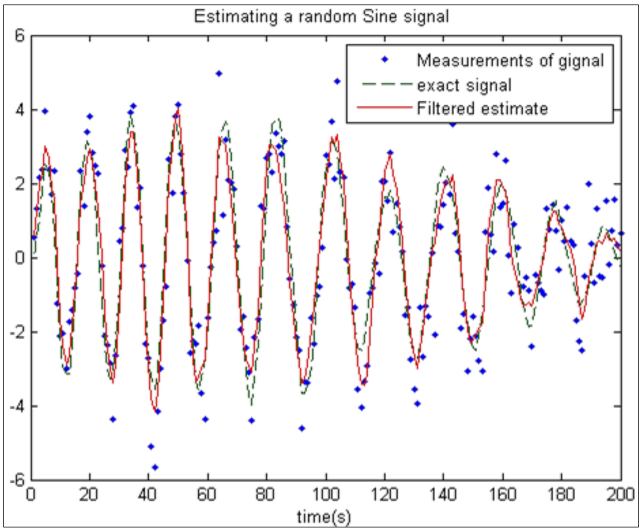

Figure 18 Distribution of measurement with estimation of signal

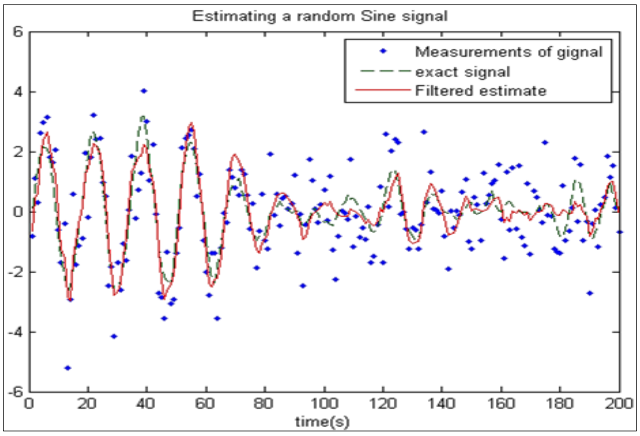

Figure 19 Distribution of measurement with estimation of signal with more noise variance

6. Conclusion

In this article, we have presented adaptive KALMAN filtering algorithm, which is based on recursively forming approximations to the joint distribution of state and noise parameters. The performance of the different variance measurement has been demonstrated in a simulated application. Then, limitation and performance of this approach in high dimensional data are simulated.

Compliance with ethical standards

Disclosure of conflict of interest

Researchers had no problems in this study.

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